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Evaluating the special role of time in the control of handwriting *

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Claims that time has a special role in the control of writing and the specific claim that writing time is absolutely invariant across changes in writing size are evaluated in two experiments. The first examined writing time for 24 undergraduate subjects who produced the string *eyleyl* with the dominant hand or arm in blocked repetitions having different vertical size targets. These variations produced small but systematic changes in writing time. The second experiment explored whether the small range of writing-time variation observed in experiment 1 was due to structural or strategic limitations. This experiment showed, for four undergraduate subjects, that writing time can be varied precisely across a wide range (0.6 to 1.66 of 'normal') while maintaining shape and vertical size constant. Taken together, these experiments suggest that, although relative stroke timing is approximately maintained, absolute timing is not critical to writing. The limited range of writing times typically observed should, rather, be ascribed to a strategic gradient that, along with other influences, broadly defines preferred writing times.

This paper also describes a new application of Generalized Procrustes Analysis of shape, and this procedure is applied to the trajectories generated in both experiments. Although several small failures are noted, these analyses generally confirmed previous claims that shape is invariant across changes in writing time, size, and writing with the hand versus the arm. This result is a necessary buttress to the conclusions just described. Shape variability was also assessed in these analyses. This variability soared as writing time was reduced from normal, but showed only a small, insignificant increase as writing time was increased from normal. There were also small, predictable changes in spatial variability across changes in size and effector.

Time has often been seen as playing a special role in the control of handwriting. This place of prominence appears to be based on two observations: (a) that there is temporal invariance across changes in writing size; (b) that the size of letters differing in size but not shape –

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e.g. *e* versus *l* – is determined by changes in timing. In addition, research and several theories for the control of handwriting (e.g., Freeman 1914; Hollerbach 1981; Yasuhara 1975) suggest that the temporal invariance observation reflects a structural limitation and thus that changing writing time for fixed writing size must result in differences in the shape or spatial variability of the written trajectory. This paper looks at these claims in the context of two experiments. The first examines the effects on writing time and written trajectory shape resulting from changes in writing size and effector. The second examines the effects on written trajectory shape resulting from changes in writing time with writing size and effector held constant. Finally, since the analysis of written trajectory shapes is an important element of this research, this paper introduces new techniques for isolating and comparing shape differences in writing.

My interest in these questions arises from earlier research on effector independence in writing (Wright 1990). This work confirmed previous observations (Merton 1972; Raibert 1977) that the shape of a person's writing is strikingly similar when produced using different effectors. This similarity by itself probably should not be seen, however, as strong evidence that writing with these effectors is controlled by a single, effector-independent motor representation since the implicit task demands in these experiments may suggest to subjects that shape preservation across conditions is their primary goal. In particular, the contrast of writing with the dominant and non-dominant hands revealed large, systematic changes of kinematics, stroke decomposition, and fluency, as well as subtle changes of the shape. Similar comparisons of writing with the dominant hand and arm did not reveal such striking differences. Instead, writing with the dominant hand and the dominant arm appear to be controlled by mechanisms that share common representations lower in a hierarchy of increasing motor specificity.

The impetus for the current work grew out of the single, substantial change that Wright (1990) observed between writing done by the dominant hand versus the dominant arm: a change in movement time. Across the two kinds of writing material studied by Wright, changing from the hand to the arm resulted in a 150% increase in trajectory length and a 25% increase in writing time. From a simplified view of the 'normal' behavior of physical processes, a 25% increase in time seems small coupled with a 150% increase in size. From the perspective of absolute writing time invariance, this increase in time is quite large. There is a confounding element in these results, however, since the observed increase in trajectory length is also associated with the change of effector between the hand and the arm. Thus, based on these data alone, it is conceivable that the claims of writing time invariance apply only to handwriting or, perhaps, temporal invariance holds equally within hand- and arm-writing but not across the two effectors, in which case the argument for effector independence between the dominant hand and arm is undermined. Alternatively, perhaps the claims of temporal invariance may be too strong, in which case the relatively small increase in time associated with the change of effectors can be seen as the natural outcome of scaling a common effector-independent motor program.

Previous research on temporal invariance in writing

In his influential description of the coupled oscillator model for writing, Hollerbach (1981: 153) describes temporal invariance as 'an accepted observation in the handwriting literature'. Table 1 summarizes the studies cited by Hollerbach along with several other selected

Table 1
Selected studies cited in support of the claim of writing-time invariance.

	Manipulation & stimuli	Subjects	Replications	ΔSize	ΔTime
Denier van der Gon and Thuring (1965)	Repeated *momom*	1	1	600%	5% (ns)
Freeman (1914)	3 or 4 isolated letters. Normal/Large size by instruction	3 adults 10 children	1 1	120% 20.2%	3% 25%
Hollerbach (1981)	Alternating *el* pattern at two instructed sizes	4	3	78%	0.1%
Stelmach and Teulings (1983)	Wrote *hye* as fast as possible at two target heights: 1.5 and 3 cm. Size uncertainty.	13	40	93%	5% (ns)
Yasuhara (1975)	(1) *e*s and *l*s in three 4-letter words. (2) loops	2	Not reported	150%	0%

studies that argue for this acceptance of the null hypothesis that there is no effect on writing time of writing size.

Without examining these studies in detail, it is clear that, with the exception of the study by Stelmach and Teulings (1983; also described in Stelmach et al. 1984), these studies are small and thus may not have had the power to detect effects of the size reported in Wright (1990). In addition, the small number of replications under unusual conditions raises the concern that these results may not truly reflect normal behavior. Freeman's (1914) results for children's writing are usually not cited in this regard. These results are interesting, however, since the children's data, unlike that of the adults in this study, give no suggestion of temporal invariance.

The study of Stelmach and Teulings (1983) requires special mention. This study was designed to investigate the effects of writing-size uncertainty rather than temporal invariance. In a two-choice, reaction-time procedure, the tone to respond was used to signal which of two, unequally probable, writing sizes was the target for that trial. The data summarized in table 1 are taken from two conditions in which the small and large target sizes had an 80% probability of being selected. From the figure summarizing this experiment it is possible to observe that for each of nine measured strokes, the duration with the large size is as large or larger than that in the small size. Measuring from the figure, I have calculated the 5% increase shown in table 1. Stelmach and Teulings (1983) report that this increase is non-significant based on sign tests run separately on the durations for each segment. From this result we can infer that a 10% increase in duration across the size conditions is at least as plausible as a 0% increase. It is certainly conceivable that a substantially larger increase would also be contained within a 95% confidence interval for the mean 5% increase.

From shortly before Hollerbach's (1981) summary of the evidence favoring temporal invariance until now there have been at least three studies reported with results contrary to this conclusion. These studies are summarized in table 2. As the table shows, these studies are generally somewhat larger than those reported in table 1. However, each of these studies has elements that might lead one to question the generality of the results. In addition, all three studies are similar in that they compare data only from loops of different sizes: e.g., *e* versus *l*. The next section discusses the possibility that the mechanisms used to create changes in size between letters with the same

Table 2
Selected studies not supporting the claim of writing-time invariance.

	Manipulation & stimuli	Subjects	Replications	ΔSize	ΔTime
Wing (1980)	*l* and *e* in the string *ele* embedded in 4 of 28 words. Two instructed sizes.	24	1	27%	24%
Greer and Green (1983)	One or two *l*s or *e*s: isolated, repeated, and alternated. Measured time to peak velocity. Two target sizes in a 2:1 ratio.	8	5 of 10	Not reported	18%
Thomassen and Teulings (1985)	Loops of 7 instructed sizes from 0.25 cm to 16.5 cm written at 'maximum' speed.	12	Not reported	0.25–1 cm 1–16 cm	0% 90%

shape produced with the same overall writing size (meso-context changes in the terminology of Thomassen and Teulings, 1985), may be different from the mechanisms used to change the overall size of writing (macro-context changes in the terminology of Thomassen and Teulings, 1985). Since these experiments all compare only letter shapes whose size differences could be conceivably interpreted by subjects as changes of the macro- or meso-context, there is a possible ambiguity in the interpretation of their results.

The results of Thomassen and Teulings (1985) are of particular interest. In that study the target vertical heights for the writing were varied from 0.25 cm to 16.5 cm in seven steps. This large range of sizes virtually guaranteed that the subjects had to switch at some point from writing with their hand to writing with their arm. Unfortunately, the design of the experiment did nothing to constrain where that switch was made. The figure reporting these results clearly shows a positively accelerated relation between vertical height and duration. The authors' verbal summary of these results states that between 0.25 cm and 1.0 cm there was no increase of duration with size (the scale and logarithmic axes make it virtually impossible to measure the size of any increase in this range from this figure). This is clearly part of the size range for which subjects would normally use handwriting. For the larger sizes, for which subjects would be increasingly likely to switch to writing motions performed primarily with the arm, there is the clear increase in duration summarized in table 2. Thus the interpretation of this experiment runs into the same ambiguity as encountered trying to interpret the results from Wright (1990).

Theoretical arguments for temporal invariance

The idea that time plays a critical role in the control of writing fits well with several prominent models for how handwriting is controlled. For expository purposes I will focus on the coupled oscillator model of Hollerbach (1981), but this argument with different details could also be made based on other models. The coupled oscillator model builds on the pervasive hypothesis that shape generation in handwriting can be decomposed motorically into two, roughly orthogonal, movement components (e.g., Denier van der Gon and Thuring 1965). In Hollerbach's model, the velocity of each of these components is determined by an oscillator. These velocities are superimposed on a constant rightward velocity. Hollerbach demonstrates that the oscillators may be based on any of a large class of oscillation patterns, although for simplicity much of this development is based on sinusoids. For a given oscillation pattern, the action of each oscillator is determined by its amplitude, frequency, and phase parameters. Although possible, in principle, to control shape by changing the relative frequency of the oscillators, this is difficult and leads to problems maintaining an even baseline; thus it appears that the frequency parameters of the two oscillators are kept the same. Hollerbach has shown that by modulating the remaining free parameters, this system can be made to produce connected shapes characteristic of Palmer script.

As Hollerbach (1981) shows, there are two strategies for modulating size by adjusting the oscillation parameters. One strategy begins by scaling the amplitude of the vertical oscillator to achieve the desired height. However, just altering this value is not sufficient, since, although writing shape is roughly preserved, the slant changes dramatically: larger figures also would become more vertical. To compensate for this, the relative phase of the two oscillators and the amplitude of the horizontal oscillator must also be adjusted. The second strategy starts by changing the frequency of both oscillators. This approach has the advantage that slant is not affected. However, to maintain a constant shape it is also necessary to adjust the horizontal amplitude.

The first of these strategies leaves stroke timing invariant as is required by the temporal invariance claim. Hollerbach (1981) argues that it is primarily the second strategy that is used to modulate size within a word: e.g., between *e* and *l*. Thus, he reports that an *e* written in large writing may be taller than an *l* written in small writing and yet still be written in less time.

This association of movement time (or frequency) with shape, independent of size, suggests the possible interpretation that it may be difficult for the writing control system to determine the phasing and amplitude parameters necessary to produce a given writing shape with a constant height and slant and an unusual writing speed. Yasuhara makes an explicit version of this claim:

> 'Different strokes are caused by the different timing of application of the force. On the other hand, the magnitude of the force used determines the *general size* of the word. Thus our supposition for fast handwriting is that the shapes of letters and words are coded only in time, it is the duration of the muscle contraction that is coded, not the magnitude of the force used'. (Yasuhara 1975: 243–244)

From this perspective, temporal invariance across size changes may occur because the control system is incapable of changing durations. A more plausible interpretation from this perspective, however, is that temporal invariance is necessary if shape invariance is to be maintained across changes in writing size. Either way, however, temporal invariance is seen from this perspective as a *structural* constraint.

A weaker version of this hypothesis, consistent with the conclusions of Freeman (1914) and others since then, is that, although the writing control system can adapt to unusual speeds, the resulting writing is less stable and of lower quality. Interpreted within the context of the coupled oscillator model, this decrease in quality and stability may reflect control parameters that are less finely tuned for writing at unusual speeds and that, perhaps, are also less stable. This instability, along with speed–accuracy tradeoff effects that may become important when writing at faster than normal speeds, could combine to create increased shape variability rather than overall changes in writing shape when writing is done at anything but the normal speed. From this perspective, temporal invariance would seem to be a *strategic* constraint adopted by the writing control system.

Although there are apparently no studies that directly examine the flexibility and precision with which subjects can change writing speed, there are clear indications in the literature that, at least within a limited range, such changes are possible. For example, Viviani and Terzuolo (1980), arguing that the stroke timing of handwriting varies proportionally with overall changes in duration, display examples in which the overall change in duration is greater than 2:1. Although that study does not attempt to demonstrate that the shape of the written trajectory remains invariant across this large span of durations, it seems likely that had there been gross shape changes these would have been noted. This suggests that temporal invariance is a strategic constraint and that it might be useful to look for 'costs' incurred at writing speeds other than a subject's preferred writing speed.

Overview of the experiments

The rest of this paper reports two experiments run to explore these issues. The first of these experiments looked at hand- and arm-writing in which subjects were given no instructions or feedback about their writing time but were constrained to write at several different sizes. This experiment was designed to assess both the claim of temporal invariance across size within effectors and, in a controlled manipulation, the possibility that this invariance holds across effectors. Unlike previous experiments that have rejected the temporal invariance claim (Greer and Green 1983; Thomassen and Teulings 1985; Wing 1980), which examined writing only for loops or the letters *e* and *l*, this experiment examined behavior in a moderately complex writing task. Unlike Wing (1980), who reported a large failure of temporal invariance using a procedure that carefully minimized subjects' exposure to the test words, the experiments reported here involve a moderate number of replications for each subject in each experimental condition. This choice allows assessment of the stability of the writing performance in each condition, minimizes the effects of an unusual writing situation on subjects' performance, and increases the precision of the mean data. Similarly, since it appeared that some of these effects might be small, 24 subjects were run using a within-subjects design to provide adequate power for discerning small effects.

The hand–arm comparison was chosen for the effector manipulation primarily because it replicates the constrast of interest in Wright (1990). In addition, however, it is important that subjects come to the laboratory already able to write fluently with both of these effectors.

The second experiment looked at normally sized handwriting with the subjects given temporal targets and feedback to encourage them to write both faster and slower than their normal speed during different blocks of trials. One goal of this experiment was to determine how easily subjects can adjust writing speed. An additional goal was to determine whether changes in writing time led to concomitant changes in trajectory shape or increases in spatial variability.

Experiment 1

Method

Subjects

The subjects were 24 right-handed Columbia College students who participated for one hour each. They were paid six dollars for their participation.

Apparatus

Subjects wrote with a stylus containing a ball-point pen cartridge on clean sheets of paper placed on the surface of a Summagraphics M1812 digitizing tablet. Except during the first two blocks for each subject (see Design section) each sheet of paper contained two full-width horizontal lines showing the desired vertical extent for that trial. The position of the stylus was sampled 110 times per second. The digitizer was connected to an AT-class computer running DOS. A program written in C controlled sampling and storage of the writing data as well as feedback to the experimenter and subject after each trial.

Procedure

Subjects wrote the string *eyleyl* once for each of the 140 trials of the experiment. For trials with a vertical size target, subjects were instructed to match the height of their writing to the space between the two horizontal lines, but not to be especially concerned if their ascenders or descenders occasionally went past or fell short of these lines. Subjects initiated trials *ad libitum* by bringing the stylus into contact with the digitizer. A trial ended when the subject lifted the stylus from the digitizer for more than 0.5 s. Except as described in the Design, the subjects were under no time pressure. They were also never given feedback about writing time.

During blocks requiring handwriting, subjects were asked to write using primarily the motion of their hands and fingers. To ensure that they followed these directions, the subjects were requested to keep their hand in contact with the paper as they wrote. During blocks requiring arm-writing, subjects were asked to write using primarily the motion of their elbow and shoulder. To ensure that they followed these directions, the subjects were requested to keep their hand off the surface of the paper as they wrote. In the past we have observed that these instructions effectively induce subjects to write with their hand or arm, respectively.

Table 3
Combinations of effector and target size studied in experiment 1.

	Vertical target			
Size (cm):	1.0	1.5	2.5	6.4
Effector				
Hand	×	×	×	
Arm		×	×	×

An experimenter, sitting with the subject, monitored each production to ensure compliance with the effector instructions and to provide feedback if the trajectory size differed by more than 10% from the target on any trial.

Design

Subjects were randomly assigned to one of two instructional groups: one group was instructed to write 'carefully', the other group to write 'as fast as you can legibly'.

Each subject wrote the string *eyleyl* 140 times in 14 blocks of 10 trials. The effector used and the size constraints were varied from block to block. In blocks 1 and 2 there were no size constraints (and no parallel, horizontal lines on the writing paper indicating a size target). These hand-free and arm-free conditions were included to familiarize the subjects with the procedures and to provide data on performance without constraints. In blocks 3 to 8, the effector to be used and the size condition were taken from the combinations shown in table 3. The order of these conditions was counterbalanced across blocks for sets of six subjects using a Latin square. Blocks 9 to 14 were the same as blocks 3 to 8 except that the order of the conditions was reversed.

Trajectory segmentation

Several of the analyses reported later build on a pre-processing stage in which each trajectory is divided into 16 segments. These segments are terminated by endpoints that are often interpreted as stroke boundaries (Denier van der Gon and Thuring 1965; Hollerbach 1981; Morasso et al. 1982; Viviani and Cenzato 1985). For the purposes of the analyses in this paper, such a strong interpretation is unnecessary. Instead, it is only necessary that the endpoints defining these segments be ones that can be located reliably across the trajectories produced by a subject and identified analogously in the trajectories produced by different subjects.

Fig. 1 shows this segmentation applied to one trajectory. In this figure, each dot represents a separate observation sampled every 9.1 ms. Thus the separation of the dots in this figure indicates the instantaneous tangential velocity. The algorithm for segmentation identifies potential segment boundaries as points with local minima in tangential velocity. The occurrence of these tangential velocity minima tends to be highly correlated with minima in radius of curvature (Viviani and Terzuolo 1982). Once the potential segment boundaries have been identified, a dynamic programming procedure picks the subset that best matches a prototype configuration determined for each subject by the experimenter.

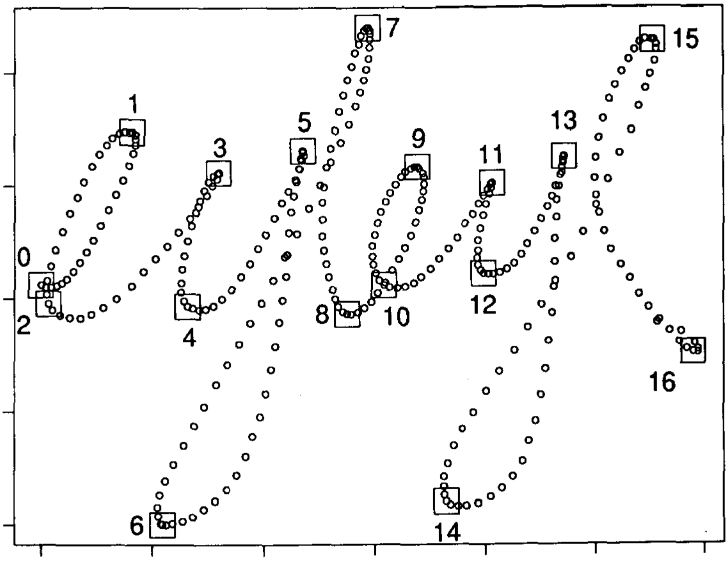

Fig. 1. A sample trajectory with marks illustrating stroke-based segmentation. Each dot in the trajectory represents a sample, equally spaced in time. The squares indicate the 17 points that demarcate the 16 segments used in later analyses. These points are chosen based on tangential velocity, curvature, and position within the trajectory.

Results

Fig. 2 shows that the average vertical extent, measured from the bottom of the descenders to the top of the ascenders, varied as a function of the target vertical extent and effector. As one might expect, the target vertical extent had a strong influence on the vertical extents the subjects produced ($F(1, 22) = 1184$, $MSe = 0.0115$, $p \ll 0.001$). However, as is typical in these experiments the increase in vertical size was not as large as the increase in the target size: the slope of this increase was 0.75 ± 0.04 cm/cm [1]. This can be seen in fig. 2 as the increasing amount that the data points fall below the slope-one, solid line with increasing target vertical extent. The slope of the increase of vertical extent did not change discernibly with instructional group, effector, or their interaction (in each case, $F < 1$). Considering only the two target vertical extents for which there is data from both the hand and the arm, there was, however, a small but statistically discernible increase in the overall vertical extent going from the hand to the arm ($\Delta = 0.15 \pm 0.06$ cm; $F(1, 22) = 26.87$, $MSe = 0.0094$, $p \ll 0.001$). The size of this increase did not depend on the instructional group nor was there an interaction of effector and instructional group (for both, $F < 1$).

[1] The notation $x \pm y$ will always be used here to indicate the mean x and its 95% confidence interval half-width y computed based on the variability across subjects.

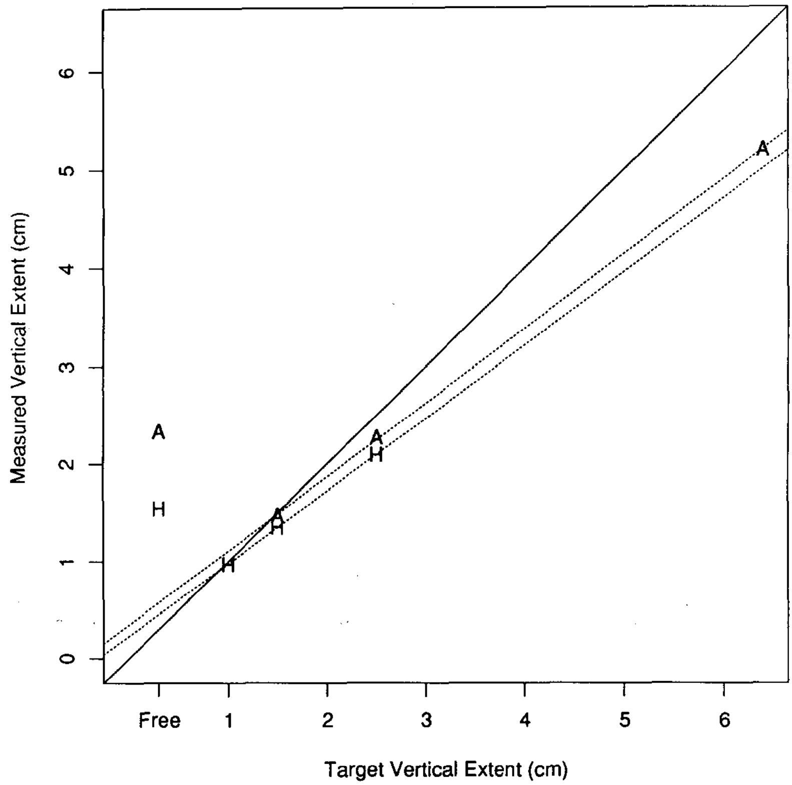

Fig. 2. Average vertical extent as a function of the target vertical extent and effector. The target vertical extent is the distance between horizontal lines on the response sheet (except in the *Free* Conditions, for which there was no vertical extent target). The average vertical extent is measured from the bottom of the descenders to the top of the ascenders. The plotting symbol *H* indicates trajectories produced primarily through handwriting. The plotting symbol *A* indicates trajectories produced primarily through arm-writing. The dotted lines are the best fitting straight lines to the average data in each of the effector conditions. The solid line has slope one and passes through the origin.

As fig. 3 shows, trajectory duration increased with increasing vertical extent. Focusing first on the six data points from the conditions with a vertical extent target, the overall increase of duration with vertical size is 0.13 ± 0.04 s/cm ($F(1, 22) = 49.82$, $p \ll 0.001$).[2] Although this slope estimate is somewhat smaller for the arm than for

[2] The results of this analysis are the same if overall trajectory length is used as the predictor rather than vertical height since these two measures of size are strongly related in this experiment: the linear function relating the average trajectory length to average measured vertical extent simultaneously for the hand and arm has an intercept of 1.3 cm, a slope of 9.77 cm/cm, and $R^2 = 0.9995$. The linear function relating trajectory duration to trajectory length has a slope of 0.015 ± 0.006.

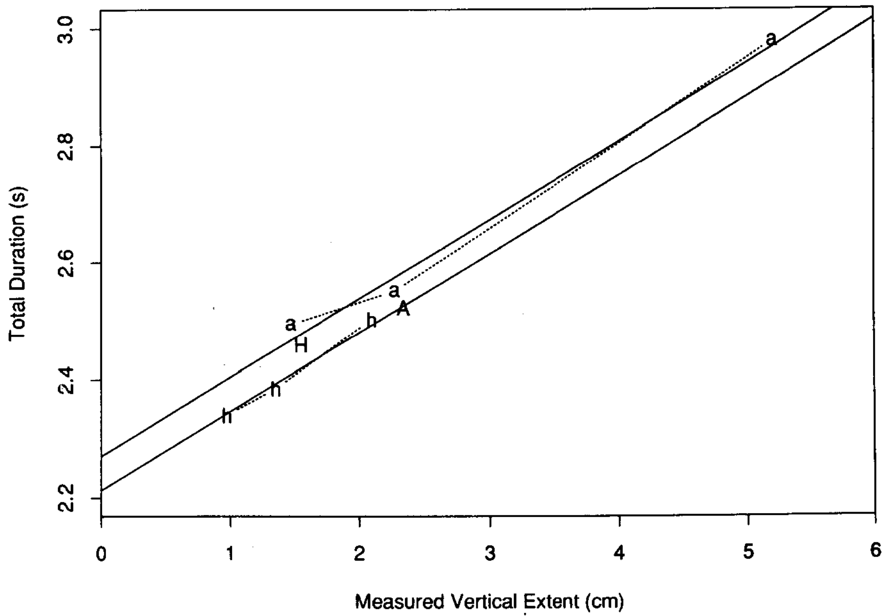

Fig. 3. Average total duration as a function of the measured vertical extent, effector, and instructional group. The plotting symbols *H* and *h* mark hand-writing trajectories. The plotting symbols *A* and *a* mark arm-writing trajectories. The lower-case plotting marks connected by dotted lines indicate data from the six conditions with vertical extent manipulated by targets. The two capital letter plotting marks indicate data from the free size conditions. The solid lines are linear functions, described in the text, fit to the hand- and arm-writing data in conditions with vertical targets.

the hand and in the careful instructional condition than in the fast condition, the effects of these factors and their interaction were not statistically reliable (in each case, $F < 1$). The intercepts for the hand and arm were 2.21 ± 0.05 s and 2.27 ± 0.05 s, respectively (for the difference between these, $F(1, 22) = 2.57$, $MSe = 0.20$, $p > 0.12$). These values also did not depend on the instructional group or the interaction of effector and instructional group (both F's < 1). This summary of the time-size relation focuses on its linear component since the quadratic trend, although apparently visible in fig. 3, is not statistically discernible. (For the hand, the quadratic coefficient was 0.056 ± 0.176 $(s/cm)^2$; for the arm, it was 0.027 ± 0.038 $(s/cm)^2$).

A comparison of the writing time in the free conditions must be interpreted carefully since the writing in these conditions varies in both effector and vertical extent. This comparison may be of some interest, however, since it reflects performance at the preferred size with each effector. Comparisons of the writing time in the free conditions with that in the constrained conditions is also of interest since this reflects the degree that having a vertical size target influenced the durations. So that the comparisons between the free and constrained writing times would not reflect differences in vertical size, an estimate of duration in the constrained conditions associated with the vertical extent produced in the free conditions was generated from the fitted linear functions. This was done separately for each subject and each effector. These values, along with the measured durations for the free size conditions, were evaluated in a three-factor ANOVA with constraint and effector entering as within-subject factors and instructional group as a between-subjects factor.

The only statistically reliable difference in this analysis was due to effector: the writing time for the arm at its preferred size was 0.105 s longer than that for the hand at its preferred size ($F(1, 22) = 9.12$, $MSe = 0.035$, $p < 0.007$). This difference is almost double that of the intercepts for the linear functions fit to the hand and arm data, since this comparison does not factor out the contribution due to the difference in writing size across the two free-size conditions. In the other important comparison in this analysis, there was no evidence that writing-size constraints changed writing time: writing time in the constrained conditions was 0.005 s longer than in the free-size conditions ($F(1, 22) = 0.01$, $MSe = 0.030$). Finally, none of the interactions of these factors were statistically discernible (all p's > 0.10).

Fig. 4 shows that the increase in duration with increasing writing size was not localized – i.e., it was not due to the addition of isolated pauses at consistent locations – but instead was fairly evenly distributed across the temporal profile of the production. Panels A and B show this for the hand and arm-writing data, respectively. Note that, in order to accentuate the effect of target size, which in these panels is relatively small compared to the changes in duration across segments, the ordinate in these two panels is normalized segment duration produced by dividing the segment duration in each condition by the mean duration across conditions for that segment. The important point in these two panels is that, to a first approximation, the three lines in each are flat and, as we would expect based on the analysis of fig. 3, the points on the dashed line dominate those on the dotted line which, in turn, dominate those on the solid line. (This is summarized by the lack of an interaction between segment and target size: for the hand, $F(30, 660) = 1.03$, $MSe = 0.00044$, $p > 0.4$; for the arm, $F(30, 660) = 1.09$, $MSe = 0.00048$, $p > 0.3$; both F's for the three-way interactions of segment, target size, and group were less than one.)

Panel C shows that the changes in duration between the hand and arm are also not due to local effects. So that this comparison is not influenced by the different vertical target sizes used for the hand and the arm, the values in this panel are the means from only the two target size conditions used with both effectors. To give a sense of the range of segment durations in these productions, the ordinate, in panel C, is unnormalized segment durations. The important aspect of the data in this panel is that, to a first approximation, the means for the arm are the same amount larger than the means for the hand in each segment (for the interaction of segment with effector, $F(15, 330) = 0.99$; for the three-way interaction including group, $F(15, 330) = 0.60$). This difference is quite stable: it holds for all 16 segments ($p < 0.0001$ by a sign test) and does not appear to change much in size. This stability may seem inconsistent with the relative instability of the intercept estimates for the linear functions fit to writing time for the hand and arm. These observations can be reconciled by noting that, within subjects, there are stable, additive differences between writing time for the

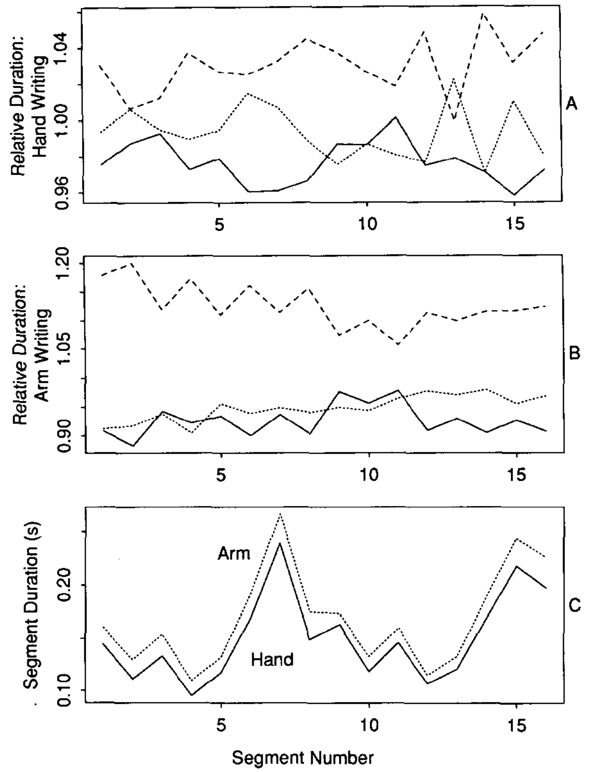

Fig. 4. Segment durations by segment and either writing size or effector. (A) The relative duration of 16 stroke segments for handwriting in each of three vertical size conditions normalized by the average across vertical size of the duration in each segment. The three line styles mark the target vertical size conditions: solid line = 1.0 cm, dotted line = 1.5 cm, and dashed line = 2.5 cm. (B) The relative duration of 16 stroke segments for arm-writing in each of three vertical size conditions normalized by the average across vertical size of the duration in each segment. The three line styles mark the target vertical size conditions: solid line = 1.5 cm, dotted line = 2.5 cm, and dashed line = 6.4 cm. (C) The mean (unnormalized) duration of 16 stroke segments averaged across the target sizes present in both effector conditions (1.5 cm and 2.5 cm). The two line styles mark the effector used: solid line = hand-writing and dotted line = arm-writing.

hand and the arm ranging in size up to 27 ms per segment. Across subjects, however, this difference is not stable with 9 of the 24 subjects writing faster with the arm and 15 writing faster with the hand.

Discussion

These data provide strong additional support for the claim that writing time varies with writing size. They show that this increase applies equally to writing by the hand

and writing by the arm with a rate of increase that is indistinguishable between these two effectors. Although within subjects there are consistent, additive differences between writing time for the hand and the arm after the writing size has been equated, across subjects the direction of this difference varies so that there is no systematic difference between effectors. Taken together, these results replicate the results of Wright (1990) for the comparison of writing time with the hand and the arm.

One concern about the generality of these results is related to the use of parallel horizontal lines to indicate the target vertical extent to the subjects. This manipulation is fairly common (e.g., Greer and Green 1983; Stelmach and Teulings 1983). However, it has been observed in the past that writing with size boundaries is slower than writing without such explicit constraints. This has led to the suggestion that writing in conditions such as those studied here is in some sense unnatural. An examination of fig. 3 reveals, however, that the durations in the free size conditions fall close to and are statistically indistinguishable from the durations expected for similar size writing in the constrained size conditions. Thus, on the whole there appears to be little evidence, at least from writing time, that the constraint imposed by the vertical extent targets in this experiment radically altered writing behavior. This discrepancy with earlier work may be due to instructions given subjects in this experiment. These instructions emphasized that the role of the parallel horizontal lines was to indicate a target for the vertical size of the writing, rather than to act as a guide constraining writing as might be the case, for example, on children's writing work sheets. These instructions coupled with the generous window on sizes allowed before size feedback was provided ($\pm 10\%$), gave subjects the freedom to end strokes below or above the target size lines. This discrepancy may also reflect a tendency in the previous work to use nominal writing size in analyses rather than to take into account the differences in writing size between conditions with matched writing size targets.

Another issue raised by these data involves the absence of effects from the instructional manipulation. Informal observation and introspection both suggest that subjects will alter the speed of their writing when instructed to write fast or carefully. During the two free size blocks in this experiment, which were the first two blocks of the experiment, there was a tendency in this direction but it was not statistically reliable: writing time was 10 ms/stroke less for the fast group than for the careful group ($F(1, 22) = 0.79$). For the conditions with vertical size targets, average durations for the two groups are virtually identical. One possibility to explain the minimal impact of these instructions is that they may not have been sufficiently salient to the subjects since these instructions were only given at the start of the experiment and were not supported by feedback during the experiment. This is consistent with the small, highly variable effect of the instructions during the initial, free size blocks and the complete absence of an effect of these instructions in the subsequent blocks during which subjects also were concerned to satisfy the size constraints. This small effect might, however, also reflect a preference by subjects to maintain a relatively constant writing time. This possibility is discussed further below.

The results here taken together with those of Wing (1980) provide a strong case that the time to write reasonably complex strings under fairly natural conditions varies with writing size both when, as was the case in this experiment, there is extensive practice with the procedures and the materials and when, as was the case in Wing's (1980) research, practice with the task and materials is minimal. The importance of this result is in its implications for the claim that changes in global writing size are accomplished solely by scaling the amplitude parameters controlling the performance not by changing timing. Although this hypothesis is appealing in its simplicity and logic, its basis is eliminated by the writing time changes observed here and elsewhere (Greer and Green 1983; Thomassen and Teulings 1985; Wing 1980).

These results also bear on the hypothesis, which motivated this research, that control of writing by the dominant hand and arm is relatively effector independent: i.e., that the divergence in control of writing by these two effectors occurs far down in the hierarchy of control mechanisms after many of the details of the movement have been 'planned'. The observation that writing time increases at the same rate for the hand- and arm-writing can be interpreted as further evidence supporting this claim. This result is certainly consistent with the interpretation that the divergence in control for the hand and the arm occurs after component durations have been specified.

Experiment 2

Although experiment 1 showed reliable increases of writing time with size, these changes were small compared to the size increases that led to them. For handwriting, a 115% increase in size led to a 7.0% increase in writing time. Similarly, for writing with the arm, a 250% increase in size led to a 19.4% increase in writing time. These results can be viewed from two perspectives. The perspective emphasized thus far is that this observed failure of temporal invariance poses problems for the suggestion that global changes in writing size are accomplished solely by changing amplitude parameters. A second perspective, however, suggests that it may be equally interesting to know why the changes in timing are as small as they are: given that subjects can change writing time, why don't they change it more freely? In particular, is the limited range of writing times observed in experiment 1 due to a *structural* limitation or was this a *strategic* choice on the part of the subjects?

Given the results of Viviani and Terzuolo (1980), described in the Introduction, or the intuitions gained from simply trying to write faster or slower, it seems likely that the limited times observed in experiment 1 are not the result of a structural limitation. If the limitation is strategic, then it becomes interesting to ask how precisely subjects can vary writing times and over what range of times variation is possible? In addition, and perhaps more importantly, what considerations lead subjects to adopt particular writing speeds and what are the costs of writing at speeds that differ from 'normal'? Several possibilities are considered below. Although it is by no means clear that the effects of these factors should be symmetric – i.e., equally applicable when writing faster or slower than normal – so that this discussion can be less abstract, these considerations are described in the context of the choice to increase writing time from normal.

Presumably one consideration would be the tradeoff of speed and accuracy: higher movement speeds are associated with reduced spatial accuracy. To maintain normal writing time when writing at larger than normal sizes requires an increase in movement speed and thus, presumably, a reduction in accuracy which may be reflected in reduced legibility. Perhaps if a high premium were put on legibility subjects would make larger changes in writing time when increasing writing size. When, as is the case in most experiments, subjects are allowed to adopt their own standards for legibility, the need to increase writing time to maintain legibility may counterbalanced by the desire, evident in most undergraduates serving as subjects, to finish quickly. Possibly consistent with this interpretation are the results of Wing (1980). In this study, the task was brief and subjects' experience with it was minimized. Together, these differences may have reduced the premium to finish quickly and thus led to the 24% increase in time associated with the 27% increase in size that Wing observed. Freeman (1914) also observed a substantial increase in writing time when his subjects were children. This experiment also involved each subject producing only a small number of tokens. Also, Freeman notes that he had a hard time getting the children to write quickly and fluently as they were so concerned about the legibility of the letters they were writing.

A second consideration working in the same direction is effort required. In general, writing at a fixed size will require less energy, and thus presumably less perceived effort, as writing time is increased.

In contrast, there are several possibly compelling reasons why subjects might not choose to write slower than normal when the task would otherwise suggest this strategy. In particular, Greer and Green (1983) suggest that shape is inextricably bound to speed. This implies that changes in writing time may cause undesirable changes in the shape of the trajectories. More subtly, and contrary to the usual expectations for a speed–accuracy tradeoff, it is possible that any change from normal writing speed increases variability of the trajectories.

Finally, there may be considerations that depend on the representation of the motor programs for writing or on characteristics of the effectors themselves. For example, Teulings et al. (1986), have argued, using a heuristic usually identified with Bernstein (1967), that since it is the spatial and to a lesser extent the temporal patterns of handwriting that are invariant across repetitions and manipulations such as size, it these characteristics that must be the descriptors in the underlying motor programs. Even if this inferred temporal component is modifiable, as the results from experiment 1 suggest that it must be, there may be limits to the range over which it can be modified or costs associated with larger modifications. At the level of the effectors one expects their properties – such as mass, natural frequency, etc. – might affect the choice of writing time.

Clearly, no single experiment could separate out, or even identify the presence of, each of these possible effects. By observing performance when subjects are required to match explicit writing time targets, this second experiment should help to differentiate some of them, however.

Method

Subjects

The subjects were four right-handed Columbia College students who participated for 1 hour each. They were paid six dollars for their participation.

Apparatus

The apparatus was identical to that used in experiment 1 with one exception. For every trial in this experiment, the sheet of paper on which the subject wrote contained two full-width horizontal lines 1.5 cm apart. Once again subjects were instructed to use these lines as a guide for the vertical size of their writing.

Procedure

These were identical to those in experiment 1. The handwriting instructions were used throughout this experiment.

Design

Each subject wrote *eyleyl* 150 times in 10 blocks of 15 trials. On each block except block 1, subjects were instructed to write the string in a particular target time. Block 1 had no time constraint: subjects were instructed simply to write the string normally at the required size. The median of the 15 durations produced by each subject in this block was used as an estimate of 'normal' writing time in the calculations that determined the time targets in subsequent blocks.

In blocks 2 to 5, subjects were given a target time chosen as 0.6, 0.8, 1.25, 1.66 of their median duration in block 1. The order of these target time conditions was counterbalanced in a Latin square across the four subjects. Block 6 had as the target time the median duration in block 1. Blocks 7 to 10 were the same as blocks 2 to 5 in reverse order.

Results

Fig. 5 summarizes the main kinematic measures for the overall trajectories in this experiment. This figure shows overall movement time, size, and within-segment peak velocity averaged across segments as a function of the relative target time. So that they could be plotted together, all three measures have been normalized by their values in the condition with target time constrained to the value produced by each subject in the free time condition.

Fig. 5 shows that, overall, the subjects were able to follow the target time instructions quite accurately. This can be seen by how closely the data (marked with the symbol T) follow the slope-one, solid line in the figure. Even for the condition with a target time 1.66 of normal, in which the performance seems to deviate most from the target, the 90% confidence interval for the time ratio extends from 1.50 to 1.67 and thus just includes the target value. Fig. 5 also shows that subjects did an excellent job of maintaining the vertical extent of their writing constant as they varied time. This can be seen by how closely the data (marked with the symbol S) fall on the horizontal line. The small increase in size with target time that can be seen in this figure is not statistically discernible from zero (slope = 0.04 ± 0.10 %size/%time, $t(3) = 1.34$, $p > 0.25$). The peak velocity data (marked with the symbol V) is compared in fig. 5 to the model in which a segment's velocity profile is simply scaled to adjust for differences in movement time. This model predicts that peak velocity should vary as the inverse of the movement time. Fig. 5 makes it clear that this simple model is

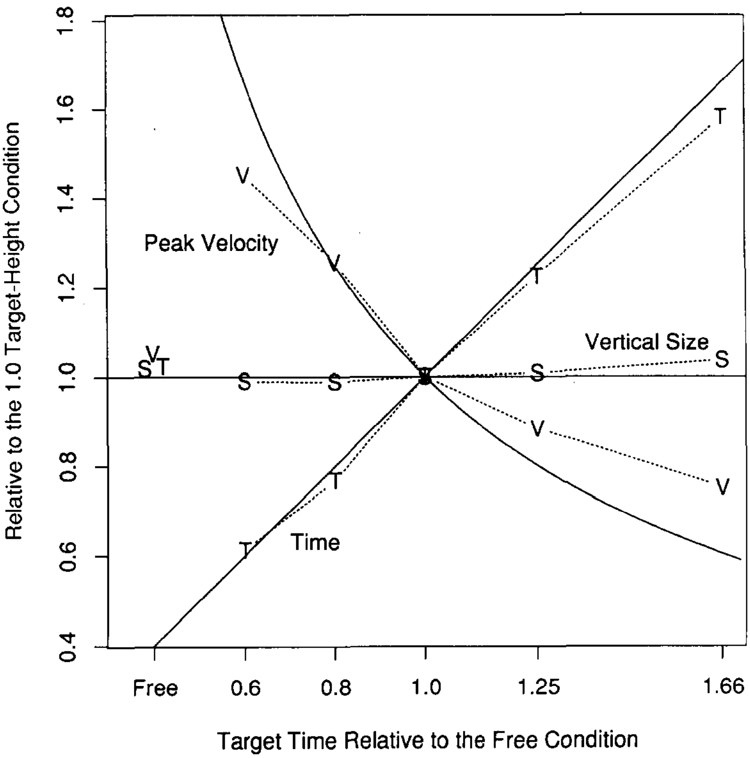

Fig. 5. Summary of three kinematic variables from experiment 2. Plotted versus target time normalized by the time in the free time condition are writing size (S), writing time (T), and the within-segment peak velocity averaged across the segments of the trajectory (V). The values of all three measures have been normalized by their values in the 1.0 constrained time condition. These values are: vertical size = 1.33 cm, movement time = 2.73 s, and average peak velocity = 16.1 cm/s. The dotted lines connect data points of each time. The solid lines represent ideal performance under simple assumptions: they all pass through the 1,1 point and have values equal to one, the relative target time, and the inverse of the relative target time, respectively.

inadequate ($F(3, 9) = 6.72$, $MSe = 0.0027$, $p < 0.02$): the peak velocity is smaller than predicted for the fastest writing and larger than predicted for the slower than normal writing. This discrepancy will be explored further in the subsequent detailed analyses.

Although fig. 5 suggests that subjects were able to adapt to new writing times, it does not indicate how fast adaptation took place. Fig. 6 shows the time course of this adaptation and suggests that subjects generally adapted to the time targets quickly during their first exposure to them and then were able to retain some of what they had learned across intervening blocks to their second exposure to each target time. Since Fig. 5 displays averages of all the data in each block, without excluding trials during the initial adaptation, subjects, after some practice, were performing even

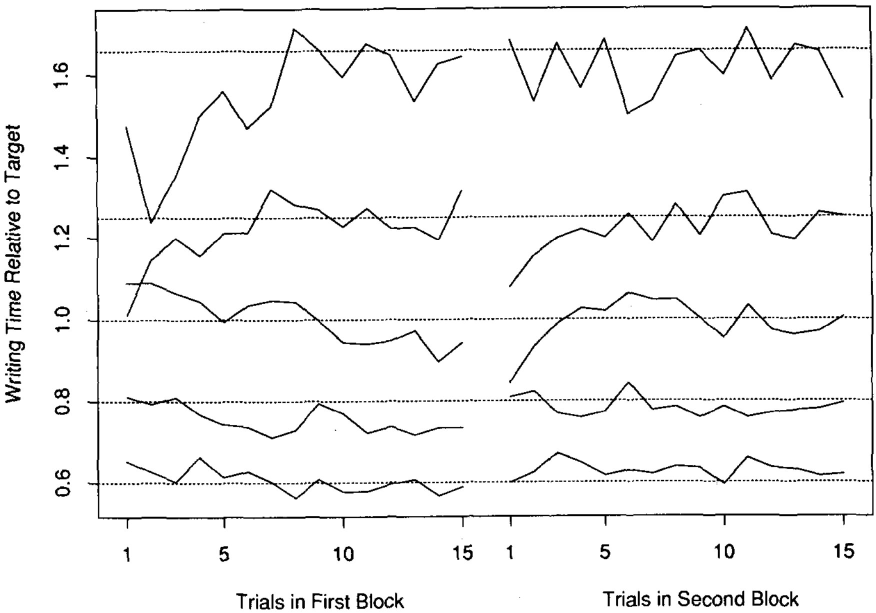

Fig. 6. Average normalized writing time by block, trial, and target time condition. Each solid line shows the writing time for a target time condition, normalized as a proportion of the normal writing time (estimated, for each subject, as the median time in block 1), and averaged across subjects. The set on the left is for the block of trials that were the first exposure to each target time and the set on the right is for the block of trials that were the second exposure. The dotted, horizontal lines indicate the relative time target in each target time condition.

better than fig. 5 suggests. Fig. 6 also indicates that subjects required more trials to adapt to the longer than normal time targets than to those that were shorter than normal. Finally, two aspects of the performance at the normal (1.0 target) time are interesting. Note first, that the data on the left side of the figure are from block 1, the condition with no time constraint, and the data on the right side of the figure are from block 6, in which the subject was asked to match a target time which was that subject's normal writing time (estimated as the median duration in block 1). Thus, fig. 6 suggests first that this normal movement time estimate might have changed somewhat if the experiment had included a different number of trials in block 1 since there was a systematic, although only marginally significant, decline of movement time across trials (slope $= -1.2 \pm 1.5$ %/trial, $t(3) = -2.66$, $p < 0.08$). Second, when subjects were given this same time as a target in block 6, they apparently required several trials to achieve the desired target (for the post hoc comparison between the first and fourth trials $\Delta = 18 \pm 20\%$, $t(3) = 2.83$).

The lack of stability in writing time during block 1 may reflect a tendency by subjects to write faster as they become more accustomed to the equipment and procedures used in the experiment. The short writing times observed in the first several trials of block 6 may be a result of the differences in instructions and feedback between blocks 1 and 6: i.e., in block 1, subjects were not given a time target or any time feedback; in block 6, however, subjects were given both. Alternatively, these initial fast times may reflect the accumulated continuation of the trend in block 1 through the intervening four blocks. Under either interpretation, however, both these observations suggest that a specific, normal writing time was neither strongly preferred nor effortlessly attained.

Fig. 5 shows that subjects were able to maintain writing size, operationalized as the vertical extent of the writing, relatively constant as they changed writing time. There are, however, other measures of size that might have been considered: e.g., horizontal extent or trajectory length. The emphasis, up to this point, on vertical extent as the measure of size rests on two points. First, vertical extent was the measure used to instruct subjects and about which they received feedback. In addition, for experiment 1, the various measures of size that were examined all changed identically as vertical extent changed (see footnote 2). In this experiment, however, that strong correspondence between variation of the various size measures no longer held. So although, as discussed earlier, the increase in vertical extent was negligible, the increase in trajectory length was much larger (slope $= 0.14 \pm 0.13$ %size/%time, $t(3) = 3.32$, $p < 0.05$). This increase is significantly larger than the increase of vertical extent with time ($t(3) = 5.23$, $p < 0.02$). This increase is also significantly larger ($t(3) = 6.63$, $p < 0.007$) than the negligible increase for horizontal extent (slope $= 0.02 \pm 0.07$ %size/%time). Thus, the length of the overall trajectory increased without an appreciable increase in the size of its bounding rectangle.

Analyses by segment

A more detailed picture of how subjects accomplished the changes in writing time is provided by figs. 7 and 8 that break the performance down by individual segments. Fig. 7 shows the duration of each segment in five of the movement time conditions normalized by the duration of that segment in the 1.0 constrained time condition. The most important result to be drawn from this figure is that the temporal changes were for the most part generalized and not accomplished through changing the durations only of isolated segments. At the same time, fig. 7 suggests that the subjects did not always change writing time by adjusting all segment durations evenly. Analyses of variance across segments for each of the target time conditions show that there were statistically discernible changes in the slowest target time condition ($F(15, 45) = 3.55$, $MSe = 0.027$, $p < 0.0005$) and in the 1.25 target time condition ($F(15, 45) = 3.10$, $MSe = 0.0080$, $p < 0.002$). In the 1.25 target time condition, this inhomogeneity is confined to a marked slowing of the final two segments; in the 1.66 target time condition, this slowing is also quite prominent, but, in addition, there are other changes that span the duration profile. The free condition exhibits a similar pattern of changes, but here they are only marginally reliable ($F(15, 45) = 1.03$, $MSe = 0.0030$, $p < 0.10$). For the 0.6 target time condition, the overall effect is again only marginally reliable ($F(15, 45) = 1.70$, $MSe = 0.0050$, $p < 0.10$). This condition reveals quite a different pattern, however: although most of the segments are not reduced as much as their 0.6 target, segments 7 and 15, which run from the bottom of the *y*-descender to the top of the *l*, reach slightly below that target. Only in the 0.8 target time condition

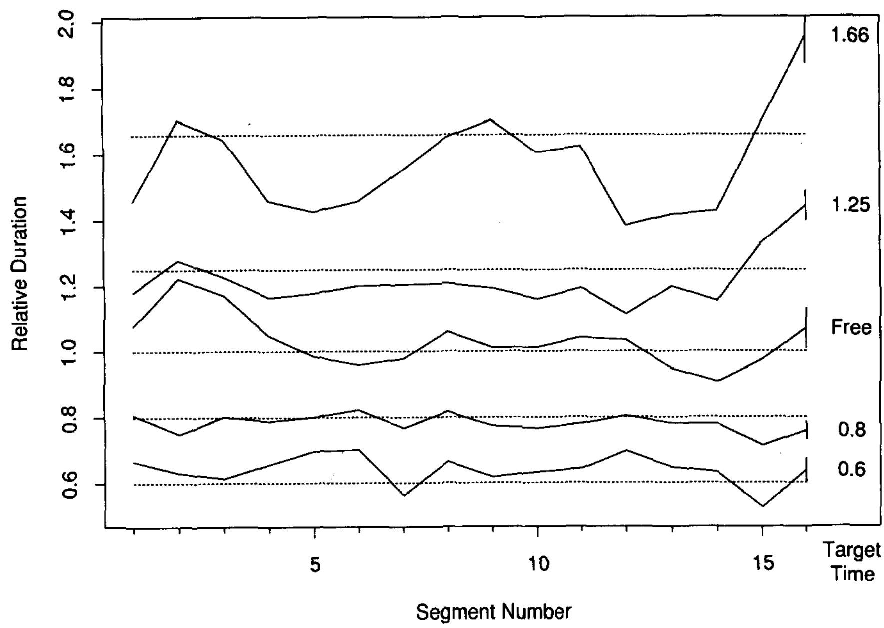

Fig. 7. Normalized duration by segment and target time condition. For each subject the duration of each segment in the constrained movement time conditions is normalized by the duration of that segment in the 1.0 constrained time condition. The figure shows these values averaged across subjects. The solid lines connect the data points in each of the target time conditions. The vertical bar at the end of each solid line has length equal to twice the pooled standard error for the means in that condition. The dotted lines mark the target values.

is there no hint of inhomogeneous temporal scaling relative to the 1.0 condition ($F(15, 45) = 1.03$, $MSe = 0.0030$, $p > 0.25$).

A second approach to characterizing how subjects change writing time is to examine whether the durations of individual segments expand and contract proportionally as the total writing time for the sequence varies (Gentner 1982, 1987; Terzuolo and Viviani 1980; Viviani and Terzuolo 1980). This was done for these data using the Constant-Proportion Test described by Gentner (1987). This test involves taking the data from all the trials for one subject and regressing the relative duration of each segment, that is, its duration divided by the total writing time for that trial, against the total writing time. If segment durations scale proportionally, are time-homothetic in the terminology of Viviani and Terzuolo (1980), then the slope of this regression should be zero. Since these regressions involve data from 120 trials, the slope coefficients estimated by this procedure are reasonably stable. Thus, once again following Gentner (1987), it is reasonable to summarize them by counting the number of these coefficients, out of 16, that are statistically reliable at the $p < 0.05$ level and comparing this number for each subject to the expectation that less than one significant coefficient per subject would occur by chance. Among the four subjects in this experiment, three subjects had nine coefficients that were significantly different from zero and the fourth had eleven. Thus, these data provide a convincing rejection of a strict proportional rescaling hypothesis. Although one might suspect from fig. 7 that this failure is primarily attributable to the 1.66 target time condition, an analysis excluding this condition produced a similar pattern of results. It should be noted that, although reliable, these deviations from proportional rescaling are generally quite small: of the 64 coefficients only 4 had absolute values greater than 0.01 – i.e., a change of more than 1% in relative segment duration for each 1 s change in total writing time – and three of these involved the final two segments. Since the difference between the average writing time in the fastest and slowest conditions in this experiment was 2.65 s, this analysis is detecting changes in relative segment duration across the target time conditions of less than $\pm 2.65\%$ in most cases and one-half of the coefficients indicate changes of $\pm 0.6\%$ or less.

An examination of the systematic changes of these coefficients across subjects does reveal several interesting patterns, however. The durations of segments 4 to 6 and 12 to 14, which make up the body of the two occurrences of *y*, all change less than the total writing times (the slope coefficients are negative). The duration of segment 1, the entry stroke for the initial *e*, also changed less than the total writing time. The entry stroke for the second *e*, segment 9, did not show this tendency, however. Instead, the duration of this segment changed somewhat more than the total writing time. This was also true of segments 7 and 15, the double strokes that go from the bottom of the *y* to the top of the next *l*, and of segment 16, but not segment 8 which is the corresponding downstroke of the initial *l*.

Segment by segment examination of the peak velocity data is complicated by the observations that relative segment duration and overall trajectory length vary between target time conditions, since variation in either of these factors might be expected to lead to changes in peak velocity. In particular, if the shape of the velocity profile is invariant, then the product of peak velocity and the ratio of the segment duration over the distance traveled normalizes for these effects and will be constant across changes in segment duration and distance. Thus, this quantity can serve as an index of the velocity-profile shape, S. [3] If the velocity profile is a pulse equal to the peak velocity throughout its duration, then $S = 1$. At the other extreme, $S = 0$ for an infinitely brief impulse whose height equals the peak velocity. $S = 0.5$ for a velocity profile with the shape of an isosceles triangle or for the positive half ($T = \pi$) of the sin function squared, $v(t) = \sin^2(t)$. In general, the value of this index decreases for profiles with sharper peaks (e.g., $S = 0.31$ when $v(t) = \sin^5(t)$).

Fig. 8 displays this index of velocity-profile shape, S, across segments for the constrained time conditions. For comparison, the raw peak velocities are displayed in the upper panel of fig. 8. Although the peak velocities increase systematically as the movement time is reduced, the pattern of change for S is more complex. The index takes different values across the segments ($F(15, 45) = 2.24$, $MSe = 0.0005$, $p < 0.02$). Even anticipating these differences across segments, one might expect little or no change across the target time conditions. Instead, S decreased systematically as the target time increased ($F(4, 12) = 17.28$, $MSe = 0.00006$, $p \ll 0.001$). Finally, there are strong interactions of segment with target time ($F(60, 180) = 3.47$, $MSe = 0.00015$,

$p \ll 0.001$). Since it is of particular interest to know for which segments this index remains relatively constant across the target time conditions, the stars, at the bottom of fig. 8, mark the segments for which there is a statistically reliable effect of the target time conditions. Clearly, a major contribution to the interaction between target time and segments comes from segments 1, 7, and 15, for which there is almost no hint of the effect of target time that occurs systematically in the rest of the segments. Segments 7 and 15 are the two instances of the double strokes that go from the bottom of the *y* to the top of the next *l*. The shape index for these segments does not change appreciably across target time although these segments show the largest changes in raw peak velocity. The value of S for these segments is also relatively small, at least compared to the values in the shorter target time conditions. The beginning and ending segments, segments 1 and 16, have values of S that are generally the smallest for each target time condition. In segment 1, as in segments 7 and 15, S does not change appreciably with target time; interestingly, however, peak velocity also does not change much for this segment. Segment 16, by constrast, is like segments 7 and 15 in having one of the largest changes in raw peak velocity across the target time conditions but differs in that S also changes across target time for this condition. Finally, ignoring these four segments, there is a tendency for downstrokes to have smaller values of S than upstrokes. This tendency is strongest for the shortest target time conditions and in the first half of the production.

[3] More formally, let $\nu(t) \geq 0$, with $0 \leq t \leq T$, be an arbitrary function defining the shape of the prototype tangential velocity profile. (Typically, this function would be zero at the endpoints, $\nu(0) = \nu(T) = 0$, although this is not necessary here.) Then the requirement that a set of velocity profiles have invariant shape across changes in distance and time, implies that they all come from the family of functions $\nu(t \mid p, r) = p\, \nu(r\, t)$, where $p > 0$, $r = T/T'$, and T' is the segment duration. Without loss of generality, the prototype function can be defined such that $\max[\nu(t)] = 1$, in which case p is the peak value of the generalized velocity function. In addition, let $D(p, T') = \int_0^{T'} \nu(t \mid p, r) \mathrm{d}t$ represent the distance traveled in a segment with the velocity profile $\nu(t \mid p, r)$; the special case $D(1, T)$ is the distance traveled during a segment with the prototype velocity profile. Calculus gives the result that

$$D(p, T') = D(1, T)\frac{pT'}{T}.$$

The velocity profile shape index, S described in the text, is defined using this notation as $D(p, T')/pT'$. For all velocity profiles in the family derived from a single prototype, $\nu(t \mid p, r)$, S is constant since, rearranging the equation above results in

$$S = \frac{D(p, T')}{pT'} = \frac{D(1, T)}{T}$$

and the terms in the rightmost ratio are both constants. Although based on the acceleration profile rather than the velocity profile, the force-efficiency factor, E, described by Teulings et al. (1986) is quite similar in conception and motivation to S.

Discussion

The principal result from this experiment is the demonstration that subjects are able to vary writing time freely and accurately while maintaining writing size constant, at least when writing size is operationalized as horizontal or vertical extent. One important implication of this result is the perspective it provides to interpret the relatively small changes in writing time in experiment 1 that resulted from changing writing size. Given that subjects can change writing time freely, the relatively stable writing times in experiment 1 reflect a strategic rather than a structural limitation: the writing control system chooses to maintain time relatively constant when changing overall writing size, a task which, in some control schemes, might reasonably be achieved by changing timing.

Taken together, the results of these two experiments suggest that there is a

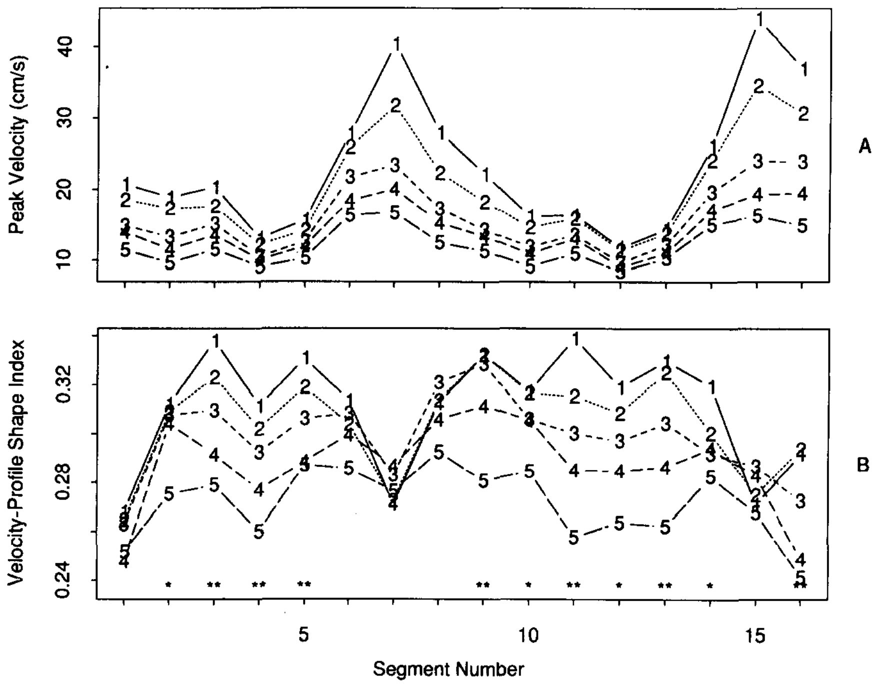

Fig. 8. The average peak velocity (panel A) and the velocity-profile shape index, S (panel B), displayed as a function of segment and target time condition for the five constrained time conditions. See the text for a description and justification of the shape index. The different numbers used as plotting symbols and the connecting lines mark the data from each of the target time conditions: 1 = 0.6, 2 = 0.8, 3 = 1.0, 4 = 1.25, 5 = 1.66. The asterisks just above the abscissa in panel B indicate the level of statistical reliability associated with the effect of target time on S in each segment: * indicates $p < 0.05$, ** indicates $p < 0.005$.

preference gradient for writing time. However, experiment 2 suggests that 'normal' writing time does not represent a strongly preferred point on this gradient. Support for this hypothesis comes from fig. 6. This shows first that writing time in the free time condition systematically declined over the 15 trials in that initial block. In addition, when, in block 6, subjects were asked to reproduce the median writing time from block 1, there was a discernible adjustment process before this 'normal' target was again achieved. Assuming that there is validity to the operational definition for 'normal' writing time used in this experiment, neither of these observations seems consistent with the idea that, to use a mechanical analogy, the speed control has a detent labeled 'normal'. In other terms, there seems to be no evidence for a strong attractor at a point associated with 'normal' writing speed in the control topology of this writing behavior.

The evidence from this experiment suggests that the strategy used to change time was neither of two simple alternatives: i.e., proportional changes to the duration of all segments or, as in speech, localized changes to easily modifiable segments. As described by Terzuolo and Viviani (1980; Viviani and Terzuolo 1980), the durations of all segments varied as total writing time changed. Deviating from their characterization, however, the results of this experiment exhibit clear, if small, deviations from strict proportional rescaling of the segment durations. Thus these results are consistent in this regard with those of Wing (1978) and Hollerbach (1981, as reanalyzed by Gentner, 1987).

The relative slowing for the final segment, apparent in fig. 7 especially for the longer target times, may reflect a strategy subjects used to time their productions. Several subjects appeared to used subvocal counting to estimate the total writing time. Using this, or any other method separate from the actual production, to time the overall interval could reasonably lead to a strategy in which subjects write slightly faster than required for most of the segments and then slow down as necessary during the final segment so that the writing time coincides with the estimated interval. The possibility of this strategy complicates interpretation of these data. At the same time, if subjects are using this strategy, this implies substantial flexibility to change writing speed during the production.

No matter how the segment durations change as the overall writing time changes, any change in a segment's duration requires a modification of its velocity profile if the total distance traveled is to remain constant. For example to travel a fixed distance in shorter time either the amplitude of the entire velocity function must be increased or its shape must be changed so that the velocity is closer to the peak velocity for more of the interval. Since roughly symmetric, single-peaked velocity profiles are generally observed for strokes in writing and for aimed movements in general, one might hypothesize that the velocity profile is changed by scaling it in amplitude and duration without changing its shape. This hypothesis has been proposed for aimed hand movements (Meyer et al. 1982; Schmidt et al. 1979). Fig. 8 shows that while this control strategy appears to hold for segments 1, 7, and 15, for most of the segments a change in time also implies a change in the shape of the velocity function. This change is in the expected direction: reductions in movement time are associated with velocity functions that have larger values of S, that is, velocity functions that are less strongly peaked and more like rectangular pulses.

Although results from a single string are insufficient to generalize broadly, it appears that the segments that are atypical because of the shape invariance of their velocity functions fit within larger patterns of structural effects related to shape requirements. These patterns are reflected in systematic changes of timing, overall velocity, and the shape of the velocity function. A key element to this interpretation is the hypothesis that it is the spatial precision required of a stroke that mediates these changes. Several aspects of a stroke's shape may increase its precision requirement and thus increase its difficulty. One example is the necessity to retrace part of a previous stroke as in the letters *c* and *k*. Similarly, strokes that must reach or pass through a target require finer control. This may take two forms. In letters such as *f* and *p*, it is necessary to close an area by bringing a curved stroke back to a previous stroke. More generally, to achieve a straight baseline, strokes to the baseline must all end at approximately the same vertical location. A similar constraint is also imposed by the mid-line, ascender line, and descender line; however, in most people's writing it appears that a more lenient precision standard holds for these locations. Finally, the degree of curvature or speed of curvature change may be a factor. By this criterion, the letters *l*, *e*, and *i* stand in a progression of increasing difficulty. In relation to this it is interesting to note that there are cusps at locations where the other precision requirements are strongest – at the join between two strokes when the second must start by retracing the first and at points where one stroke comes back to join a pre-existing stroke – suggesting that these cusps, formed with complete stops and changes of direction, are a device for producing precision in direction and location. Without a substantially larger corpus it is impossible to explore how these factors combine. Even within the confines of the present data, however, we can see evidence of them at work.

Segments 7 and 15 are both upstrokes that complete the descender of a *y* and continue on to the top of the following *l*. The duration and length of these segments are both about twice as long as those of the other 14 segments, prompting the speculation that they consist of two strokes that, because of their similarity in direction, have coalesced. The long looping shape of these segments and the absence of a requirement to stop at a predefined position combine to minimize the precision requirements for this stroke. It appears to be the combination of these factors that allows the control system to adjust freely the timing of these segments by scaling the velocity function without changing its shape. Thus, these segments have no statistically discernible change in S, and the peak velocity changes more in these segments than in any others. At the same time, the duration of these segments changes more than a proportional apportionment of the change in overall movement time would predict. In a similar vein, these two segments are the only ones with durations less than or equal to the target in the 0.6 target time condition.

By contrast, consider the two sets segments 4 and 5 and segments 12 and 13. Each of these pairs forms the cup for one of the occurrences of *y*. This is an area with high-precision requirements and strokes that form cusps. Perhaps as a result, the duration of these segments changes significantly less than would be expected under a proportional apportionment of the time changes required by the target time conditions. In addition, across the target time conditions these segments have the smallest peak velocities, the smallest absolute change in the peak velocity, and some of the largest changes in the shape of the velocity profile, S. The pattern of these observations suggests that the control system attempts to minimize timing changes for these segments. To accommodate the remaining time changes, the control system adjusts the velocity profile primarily by changing the profile shape and less by scaling the amplitude.

The presence of end effects is evident in comparisons of segments 1 and 9 or segments 8 and 16. In both comparisons, the index S is substantially smaller and the velocity function is more strongly peaked, for the segment on the end than for the analogous segment near the center of the production. Comparing the relative durations across the target time conditions, the duration of segment 1 changes substantially less than would be expected on the basis of a proportional apportionment of the overall change in writing time, but its analog in the middle of the production, segment 9, changes somewhat more than would be expected. By contrast, segment 16 changes more than would be expected and incorporates final lengthening typical of the phrase-final syllable in speech production. Its analog, segment 8, exhibits neither of these effects with its duration changing slightly less than would be expected with proportional apportionment of the change in overall writing time.

Shape analyses

Although constancy of shape has not been a large concern for previous research like that reported here, interpreting the results from experiments such as these seems tenuous without some assurance that the trajectories produced in various time- or size-constrained conditions bear more than a superficial resemblance to normal writing. This analysis seems particularly important for experiment 2 in which there already exists some evidence for shape changes: although subjects were able to keep writing size relatively constant, trajectory length increased systematically with writing time.

Even if the issue of shape invariance across experimental conditions were not of particular concern, it is clearly important to be able to compare shapes of written trajectories. The production of a characteristically shaped trajectory is, after all, the goal of an act of writing. Given this it is unfortunate that, despite the many tools available to analyze the process of writing, there is little in the way of adequate tools to characterize or compare the writing product. This section outlines one contribution to a future toolbox.

Generalized Procrustes Analysis of shape

An important precursor to any detailed set of comparisons or characterizations of writing based on shape is the ability to extract the mean shape from a set of trajectories that have all been produced to be nominally identical. Of course, random variation, in local details of the shape itself, in timing, and in the global aspects of position, scale, and orientation, will lead to differences between the trajectories in the set. The subgoals then are to identify and reverse the global transformations applied to each instance, extract abstract shape information out of the kinematics of each trajectory instance, and then compute a composite or average shape across the set. The deviations in shape of each instance from the resulting average can then be treated as an estimator of shape variability and used to test hypotheses about the mean shape. The approach that I have adapted to this problem is a method known in the literature on scaling and factor analysis as Generalized Procrustes Analysis (GPA). A thorough introduction to the theory underlying this technique is provided by Goodall (1991).

The general problem of aligning the points of a trajectory optimally with those of a second, *reference* trajectory – this reference trajectory might be a second empirical trajectory, an average of several trajectories, or an experimenter-defined prototype – is difficult because it simultaneously involves solving a transformation problem and a matching problem. The transformation problem is to identify the parameters of the global transformation for the first trajectory's coordinate system relative to that of the reference trajectory so that this transformation can be inverted. This, in itself, it not a particularly difficult problem, a closed-form, analytic solution is available for most cases of interest if the correspondence is known between the points in the trajectory and the points in the reference – i.e., if the matching problem has been solved. The matching problem involves determining the point in the reference that corresponds to each point in the trajectory to be aligned so that the distance between corresponding points is minimized. This problem is also tractable if the transformation problem has already been solved. Thus, although either problem alone will yield to traditional methods, simultaneously obtaining an optimal solution to both problems may only be possible through brute force searching of a large parameter space. In the case addressed by GPA, the problem is yet more difficult because the reference is an average shape the determination of which depends on the alignment and transformation of each of the trajectories that go into it.

To make this problem tractable, the matching problem was eliminated by doing the GPA analyses reported here on the landmarks identified in each trajectory as segment (or stroke) endpoints (see the description of Trajectory Segmentation in the Methods section for experiment 1). Thus, for this approach, a trajectory is reduced to a 17 by 2 matrix of the *xy* coordinates of each landmark. This approach has the advantage that this step eliminates the messiness of timing variability from the analysis. A danger in this approach is the implicit assumption that the landmarks used (possibly through their interpretation as stroke endpoints) incorporate the invariant shape information contained within the trajectory. Support for this last assumption comes from the work of Teulings and Schomaker (their article in this volume).

The GPA analysis can be summarized more formally as follows (see Goodall, 1991, for a more detailed development). The starting point is n $m \times p$ matrices containing coordinates for m landmarks in p dimensions (in this case, n is the number of repeated trajectories produced by a subject in a condition, $m = 17$, and $p = 2$), X_i where the index i may take values from 1 to n. The goal then is to solve simultaneously for a mean shape $\bar{X}$ and the best transformation of each replication of the trajectory onto the mean shape, minimizing the total squared distance between the transformed trajectories and the mean shape. The transformations allowed invert the uncertainties of the data collection process: translation, γ_i, isotropic scaling, β_i, and rigid rotation, Γ_i. Thus, what is required are the matrix $\bar{X}$, and n values of γ_i, β_i, and Γ_i that minimize the generalized Procrustes sum of squares, G

$$G = \sum_{i=1}^{n} \left\| \beta_i X_i \Gamma_i - 1_N \gamma_i^{\mathrm{T}} - \bar{X} \right\|^2 .$$

To make the solution of this problem unique, $\bar{X}$ is centered and constrained to have size 1. In addition, the rotations are constrained so that the baseline of the $\bar{X}$ is horizontal.

In the general case with $p > 2$, or when the solution must take account of non-homogeneity in the variance of the landmarks, GPA is an iterative algorithm. Since, for these data, $p = 2$ and the $mp \times mp$ covariance matrix derived from the transformed landmark matrices is reasonably approximated by a scaler times the identity matrix, the GPA algorithm used here was the closed-form regression procedure described by Goodall (1991).

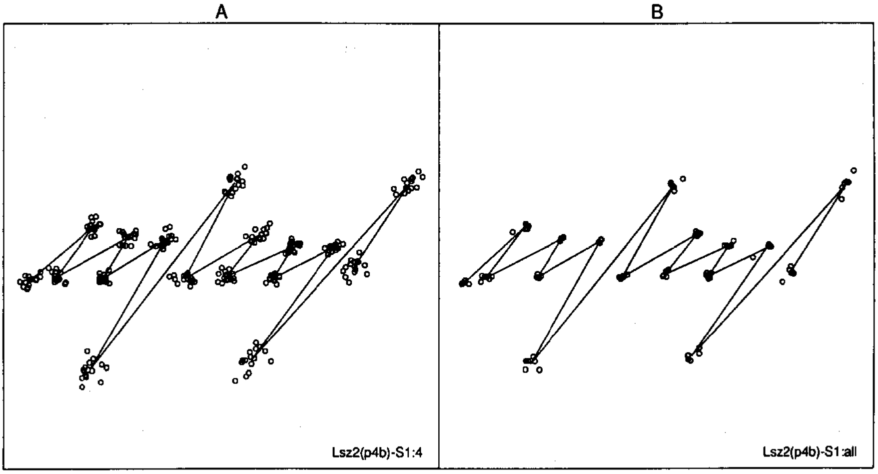

Fig. 9. An example of the two-stage application of Generalized Procrustes Analysis. The data are all from subject 1 in experiment 2. The panel on the left (A), shows the analysis for the 1.0 constrained time condition. The lines connect the landmarks in the mean shape derived for this condition. The points are transformed locations of each of these landmarks in the 15 trajectories from this condition. The panel on the right (B), shows the analysis across conditions to obtain the overall mean shape for this subject. The lines connect the landmarks in this overall mean shape. The points mark the locations of these landmarks in the mean shape computed for each of the six conditions in this experiment. The x and y dimensions have equal scales in both panels.

The GPA analysis was applied in two stages. These were always done on the data from individual subjects, since there is no particular reason to expect details of writing shape to generalize across subjects. The first stage of the analysis was done across the replications within a condition; thus, in experiment 1, across the trials within each of the target size conditions and, in experiment 2, across the trials within each of the target time conditions. Panel A of fig. 9 shows the results of this first-stage GPA for the data from subject 1 in the 1.0 constrained time condition of experiment 2. The lines in this figure connect the landmarks in the mean shape for this condition. The points mark the transformed locations of each of these landmarks in the individual trajectories that were used in the computation of the mean shape. The two points to observe in this panel are that the analysis is largely successful – the landmarks from this set of trajectories can be made to line up closely – and that the distribution of deviations around the mean trajectory are fairly similar in the x and y dimensions and across landmarks. The sum of the squared distances of each landmark in the transformed trajectories from the analogous landmark in the mean trajectory is the generalized Procrustes sum of squares, G. In this stage of the analysis G has a natural interpretation as a measure of shape variation between repetitions.

The second stage of the analysis starts with the mean shapes computed for a subject in each condition: in effect, the $\bar{X}$ matrices derived across conditions in the first stage become the X_i matrices of the second stage. A second GPA is done to determine the overall mean shape across conditions. Panel B of fig. 9, illustrates the application of this second analysis stage, again for subject 1 in experiment 2. Here, the lines connect the landmarks in the overall mean shape. The points now mark the transformed locations of the landmarks from the mean shapes derived in each of the six conditions of this experiment. (Thus, one point in each of the clusters in panel B corresponds to the locations marked by the intersections of the lines in panel A of this figure.) Clearly, the mean shapes for the six conditions correspond quite well.

Just as G for the first stage of the analysis estimates the sum of squares for the residual error, so also G for the second stage estimates the sum of squares due to differences between the conditions. Goodall (1991) shows that these estimates are independently distributed with χ^2 distributions and that they can be used to construct an F ratio to test the hypothesis of no difference between the conditions. For the case shown in panel B of fig. 9, although the differences between conditions are small, they are all statistically significant with $p < 0.05$. Clearly these tests are quite sensitive.

As described, this method has several limitations. First, as previously discussed, this analysis is based on only the location of the segment boundaries. For the purpose of visualizing differences between conditions, this limitation can partially overcome. In subsequent figures showing shapes from experiment 2, five points intermediate between each pair of segment endpoints will also be included. The locations of these points were calculated by going back to the original trajectories, transforming them according to the parameters estimated as part of the GPA, and then dividing each segment into six parts of equal length. The locations of the junctions between these parts were then averaged across replications to obtain the points used in the figures. [4]

[4] This subsequent analysis has not yet been done for the trajectories in experiment 1.

The second limitation involves the tests for differences in shape between conditions. These are large, omnibus tests and, as the example above suggests, quite sensitive. This combination makes the results of these tests hard to interpret.

This leads to the third and most serious limitation of this method. The GPA can extract the underlying shape from a set of trajectories and provides the basis for comparing the mean shape from several sets of trajectories. Clearly, this is a useful and necessary first step if shape is to be used seriously in the description of writing. The shape of writing, however, is a complex, multivariate concept. Even if the 34 variables of the mean shapes extracted using the GPA capture all the underlying shape information in a set of replications, these variables do little to describe or summarize the mean shapes or their differences between conditions. For insight at this level it is still necessary to visualize the shapes extracted and then develop ad hoc measures that capture and quantify the elements of these shapes that appear to be of interest.

Shape analyses: Changes in segment curvature

Fig. 10 shows the mean shapes produced by subject 3 for the five constrained time conditions in experiment 2. As previously discussed, the lines marking each mean shape in this figure include five points interpolated at equal distances within each segment. So that differences between conditions are more visible, the display of the five conditions has been divided across the two panels of this figure. To provide a frame of reference, the mean shape in the 1.0 target time condition is shown in both panels. Panel A, on the left, displays the shapes for the middle three target time conditions: 0.8, 1.0, and 1.25. Panel B, on the right, redisplays the shape for the 1.0 target time condition along with the shapes for the two extreme target time conditions: 0.6 and 1.66.

Fig. 10 shows that the mean shapes are quite similar for subject 3 across the target time conditions as were those for subject 1 shown in panel B of fig. 9. This way of displaying the mean shapes also makes it easier to see that there are small, systematic differences between the conditions. This display may not, however, make it easier to identify the form of those differences. The reason for this it that the least squares criterion used in the GPA to align the shapes is optimal for determining whether differences exist between shapes, but it usually does not produce alignments that emphasize the structure of those differences. Thus, it is often necessary to explore other ways of aligning the shapes derived by the GPA when the goal is to identify how shapes differ.

For experiment 2, the earlier analyses have already provided a hint about one form of the shape differences between the conditions. This is the observation that the average length of the trajectories increases with increasing target times even though the horizontal and vertical extents of the trajectories do not change appreciably. This observation suggests that there may be differences in the curvature of the segments across the target time conditions. Looking at fig. 10 from this perspective, it is possible to see that this might be true, but this is difficult since the lines representing a segment across the target time conditions do not pass through a common point. Fig. 11 makes this visual comparison easier. This figure shows enlarged versions of segments 9 through 12 from panel B of fig. 10. These are the segments that make up the second *e* and the first cusp of the bowl of the second *y*. To make it easier to see the differences in curvature across

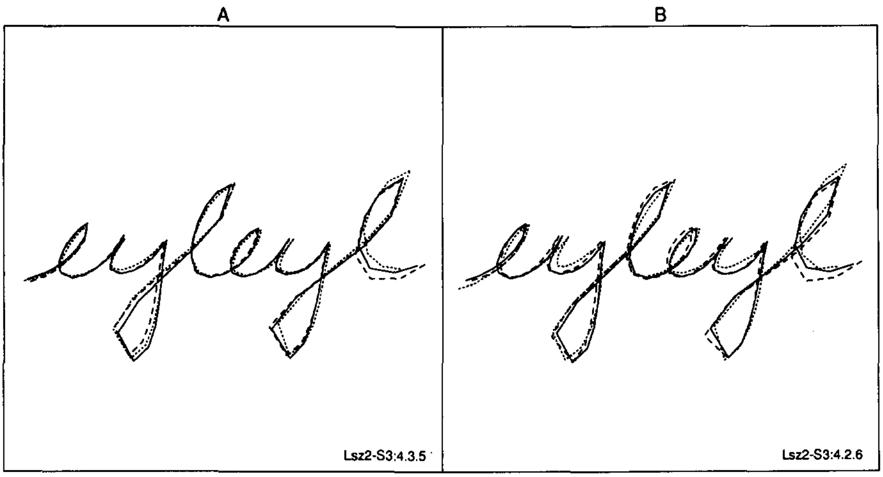

Fig. 10. Comparison, across the target time conditions in experiment 2, of mean shapes with five interpolated points for subject 3. The panel on the left (A) shows data from the middle three target time conditions: solid line = 1.0, dotted line = 0.8, dashed line = 1.25. The panel on the right (B) shows data from the two extreme target time conditions along with, for comparison, the data from the 1.0 condition: solid line = 1.0, dotted line = 0.6, dashed line = 1.66. The x and y dimensions have equal scales in both panels.

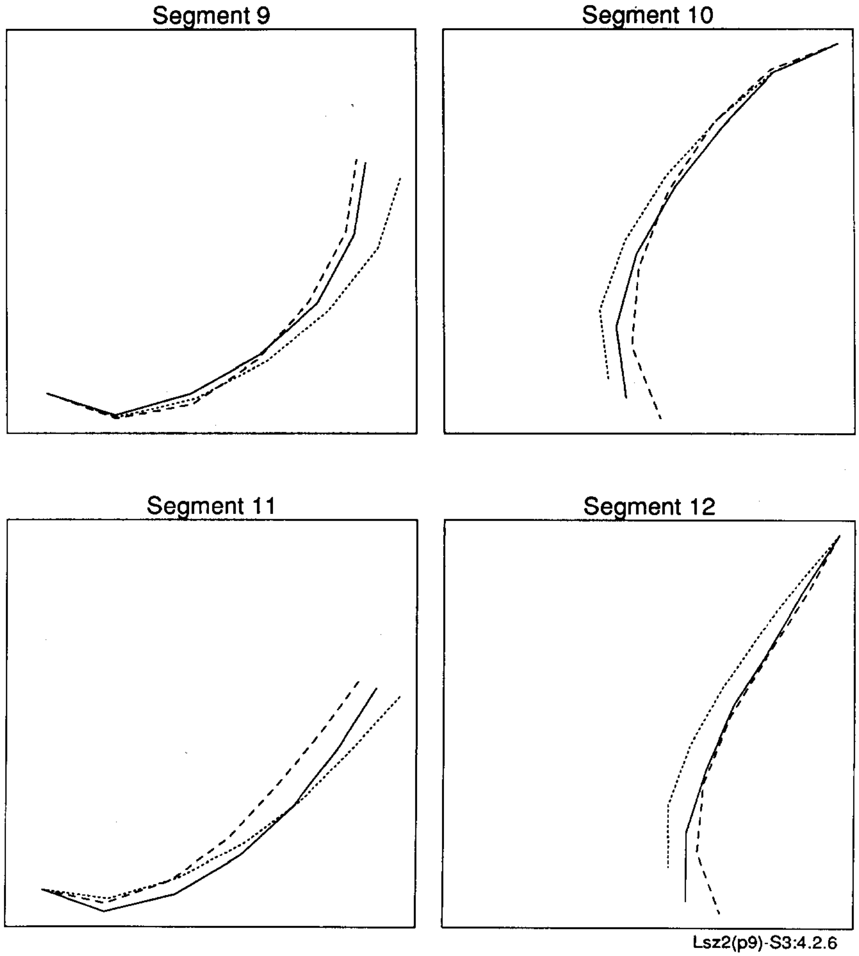

Fig. 11. Comparison of segments 9 through 12 for subject 3 in the 0.6, 1.0, and 1.66 target time conditions of experiment 2. To simplify visualization of the changes in curvature across target time, the segments in each panel have a common origin. The different line types mark the data from each target time condition: solid line = 1.0, dotted line = 0.6, dashed line = 1.66. Although each panel has a different scale, the x and y dimensions in each panel have equal scales.

conditions, the lines representing each segment have a common origin for the three target time conditions. Viewed in this way, it is clear that, for each segment, the dotted line representing the 0.6 target time condition is straighter than the solid line representing the 1.0 target

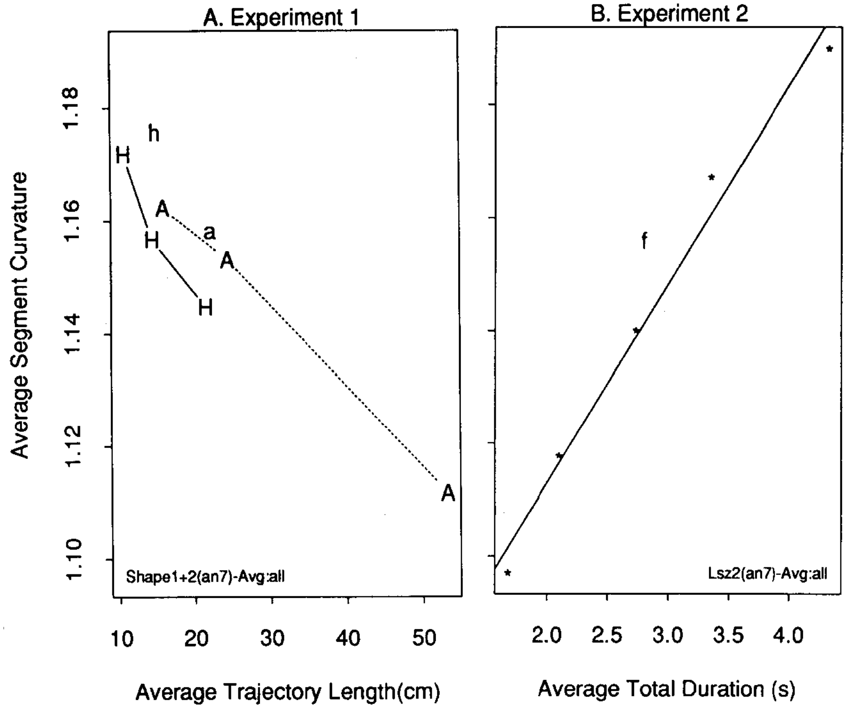

Fig. 12. Effects on segment curvature (measured as the path length along a segment divided by the distance between its endpoints) of trajectory length and writing time. Panel A displays the average curvature from experiment 1 as a function of trajectory length (the abscissa), effector (points labeled with *H* or *h* for the hand and *A* or *a* for the arm), and whether writing size was constrained (capital letters connected by solid lines indicate the constrained conditions and small letters mark the free conditions). Panel B displays the average curvature from experiment 2 as a function of writing time (the abscissa) and whether the writing time was constrained (points marked with asterisks) or free (the point marked with an *f*). The straight line in panel B represents the straight line that fits the data from the constrained time conditions best.

time condition. That line, in turn, is straighter than the dashed line representing the 1.66 target time condition.

To quantify this tendency, curvature across a segment was estimated as the ratio of the path length of the segment to the distance between the endpoints of the segment. This measure has a minimum value of 1 when the segment is straight. If a segment were a perfect half-circle, then this measure would be $\pi/2 = 1.57$; for a segment that was a perfect quarter of a circle, this measure would be $\pi/2\sqrt{2} = 1.11$. Fig. 12 shows this measure averaged over segments and subjects for experiments 1 and 2. Panel B shows that the systematic effects of target time on curvature are present in the data averaged across subjects and segments ($F(5, 15) = 18.74$, $MSe = 0.00024$, $p \ll 0.001$). A straight line with an intercept of 1.04 ± 0.12 and a slope of 0.037 ± 0.023 summarizes the data from the constrained time conditions reasonably well (for the average data, $R^2 = 0.98$).

Changing writing size also has an effect on curvature as shown in panel A of fig. 12. These data are reasonably well fit by a straight line with intercept 1.189 ± 0.025 and a slope of -0.0019 ± 0.0009. There is not a statistically discernible change in the slope or intercept between the hand and the arm (for the intercept, $F(1, 23) = 1.42$, $MSe = 0.053$, $p > 0.25$; for the slope, $F(1, 23) = 2.74$, $MSe = 0.00025$, $p > 0.10$). The direction of this effect is for the segments to become straighter as the writing becomes larger.

Shape analyses: Changes in aspect ratio

Fig. 13 shows the mean shapes for one subject in experiment 1. (The shapes in this figure appear more schematic because they do not

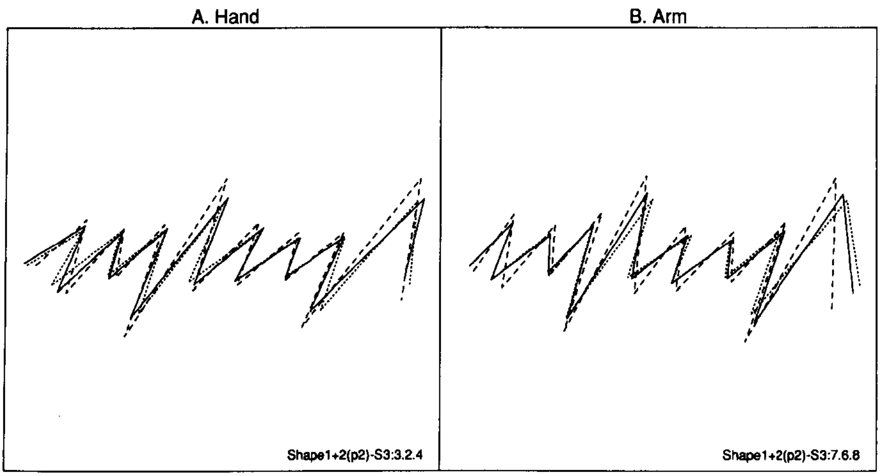

Fig. 13. Comparison of mean shapes produced by subject 3 with the hand and arm in 3 target size conditions from experiment 1. The panel on the left (A) shows shapes produced with the hand in the three target size conditions: dotted line = 1.0 cm, solid line = 1.5 cm, dashed line = 2.5 cm. The panel on the right (B) shows shapes produced with the arm in the three target size conditions: dotted line = 1.5 cm, solid line = 2.5 cm, dashed line = 6.4 cm. The x and y dimensions have equal scales in both panels.

include the interpolated points that were in fig. 10.) The two panels of this figure display the results from the three constrained size conditions done with the hand and the arm. This subject's results illustrate two observations that are generally true across all 24 subjects. First, the mean shape differences between the hand and the arm are generally small and irregular across subjects. There are, however, systematic changes in shape for writing of different sizes. Along with the changes in curvature, just discussed, there were large changes in aspect ratio. Notice in both panels of fig. 13 how the endpoints of the dashed lines, representing the larger target size conditions, are out-

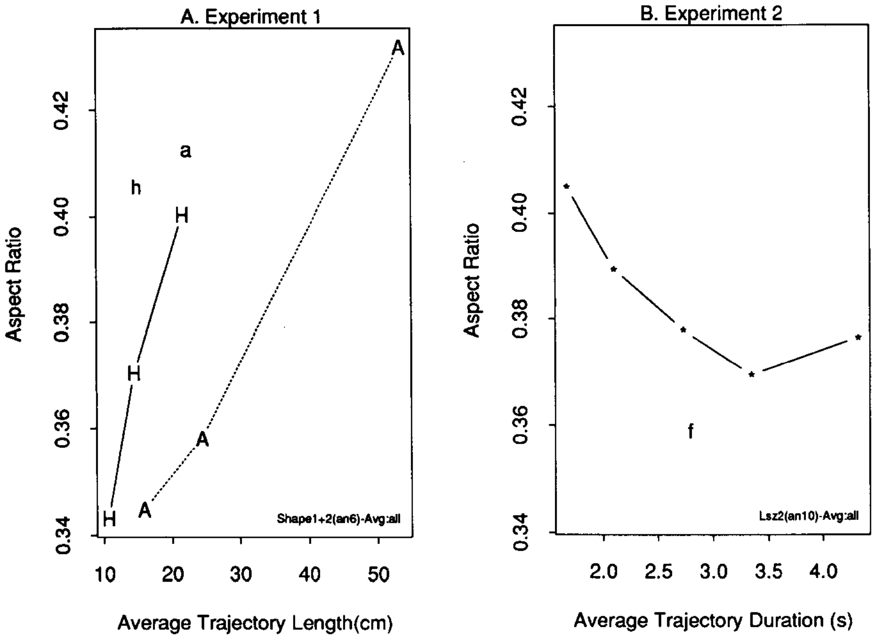

Fig. 14. Effects on trajectory aspect ratio (measured as the ratio of the rms variation in the vertical dimension of the landmark locations in a trajectory to the rms variation in the horizontal dimension) of trajectory length and writing time. Panel A displays the aspect ratios from experiment 1 as a function of trajectory length (the abscissa), effector (points labeled with *H* or *h* for the hand and *A* or *a* for the arm), and whether writing size was constrained (capital letters connected by solid lines indicate the constrained conditions and small letters mark the free conditions). Panel B displays the aspect ratios from experiment 2 as a function of writing time (the abscissa) and whether the writing time was constrained (points marked with asterisks or free (the point marked with an *f*). The straight line in panel B represents the straight line that fits the data from the constrained time conditions best. For comparison, the two panels have the same ordinates.

side of those of the solid lines in the vertical dimension. Similarly, the endpoints of the dotted lines, representing the smaller target size conditions, tend to be within those of the solid lines. The change in the aspect ratio shows up this way in the alignments computed by the GPA, because the total squared error is minimized by transforming the shapes so that their horizontal components are approximately aligned.

One measure that can be used to assess these changes in aspect ratio is the ratio of vertical variation to horizontal variation of the landmark locations in each trajectory. Fig. 14 displays this measure as a function of writing size for experiment 1 and as a function of writing time for experiment 2. Panel A displays the aspect ratio computed for experiment 1 as a function of writing size and effector. The results from the free size conditions, clearly differ from those of the analogous effectors in the conditions with constrained sizes. Excluding the data from these two conditions, aspect ratio appears to vary as a function of writing size with different rates of change for the two effectors. Linear functions fit separately to the data for each effector have intercepts that are not statistically discernible ($F(1, 23) = 0.18$, $MSe = 0.0047$). The least-squares estimate of this common intercept is 0.29 ± 0.04. The slope of the straight line fitted to the data for the hand is 0.0053 ± 0.0021 and for the arm 0.0027 ± 0.0008. The difference between these slope estimates is statistically reliable ($F(1, 23) = 9.42$, $MSe = 0.000017$, $p < 0.006$). In both cases, the effect can be interpreted as an increase in the relative variation of the vertical dimension as the size of the writing increases.

Panel B of Fig. 14 displays aspect ratio versus writing time for experiment 2. The change in the aspect ratio across the writing time conditions is relatively small, non-monotonic, and only marginally reliable statistically ($F(5, 15) = 2.63$, $MSe = 0.00039$, $p < 0.07$).

Shape analyses: Residual shape error

The final aspect of the results from the shape analysis does not concern shape *per se* but stability of shape across the trajectories produced under different combinations of effector, size, and time. This is of particular interest in the context of experiment 2, since one possible cost of changing writing time is an increase in spatial variability. This analysis is based on the generalized Procrustes root mean squared error. This can be computed from G, the generalized Procrustes sum of squares minimized during the GPA analysis, by dividing it by its degrees of freedom and then taking the square root. Note that this variability measure is computed on the transformed trajectories, which have been scaled to the common size of the mean shape. Thus it is appropriate to compare this measure across conditions involving writing at different sizes. The units of this measure are proportions of the total size. These values are shown in fig. 15.

Focusing first on the results in panel A from experiment 1, there are definite differences across the eight conditions of the experiment

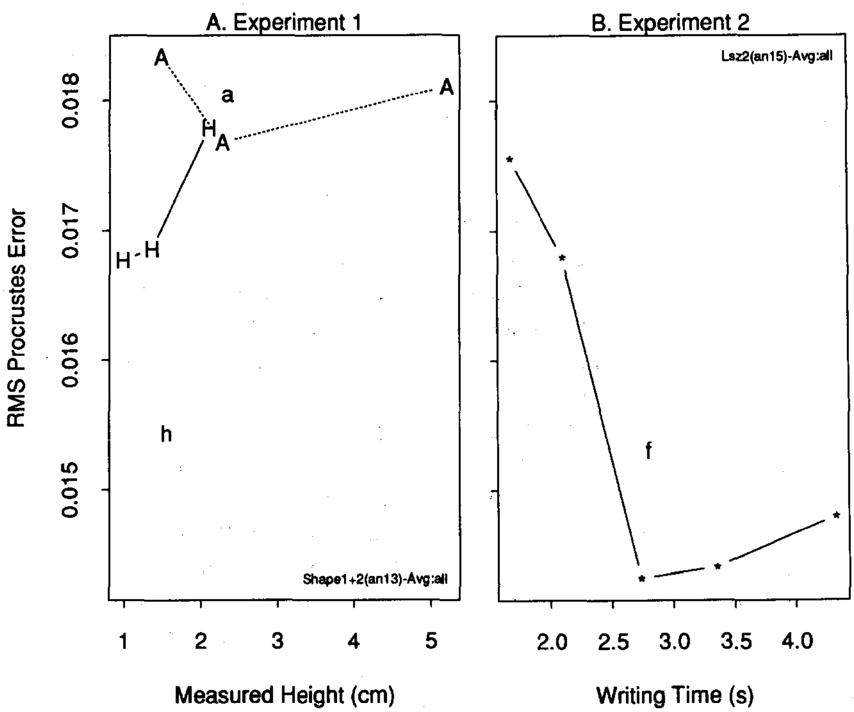

Fig. 15. Effects on shape variability (quantified as the root mean squared Procrustes shape error) of writing size, effector and time. Panel A displays the spatial variability from experiment 1 as a function of vertical extent (the abscissa), effector (points labeled with *H* or *h* for the hand and *A* or *a* for the arm), and whether writing size was constrained (capital letters connected by solid lines indicate the constrained conditions and small letters mark the free conditions). Panel B displays the spatial variability from experiment 2 as a function of writing time (the abscissa) and whether the writing time was constrained (points marked with asterisks) or free (the point marked with an *f*). For comparison, the two panels have the same ordinates.

in the level of spatial variability ($F(7, 154) = 4.17$, $MSe = 5.39 \cdot 10^{-6}$, $p < 0.001$). The Newman-Keuls procedure (Winer 1962) was used to break down the main effect of conditions. (All contrasts mentioned based on this analysis are significant with $p < 0.01$ unless specifically stated otherwise.) The clearest result from this analysis is that there was substantially less spatial variability writing with the hand in the free condition than in any other condition. Writing with the hand, variability increased as writing size increased although the difference between the 1.0 cm and 1.5 cm conditions was not statistically reliable. Writing with the arm, there appears to have been a minimum in the spatial variability somewhere around a height of 2.5 cm, since the variability in that condition was less than that in the three other conditions involving the arm. (The differences between the 2.5 cm condition and both the free condition and the 6.4 cm condition are significant with $p < 0.05$.) A set of conditions spaced more closely in height would be needed to determine the precise location and depth of this minimum. Finally, it is clear that no simple statement can be made describing how spatial variability changes with effector. Although for a height of 1.5 cm, variability with the hand is substantially less than that with the arm, for a height of 2.5 cm, their levels of variability are statistically indistinguishable. Extrapolating from these data would suggest that, at writing sizes larger than 2.5 cm, the hand would exhibit more spatial variability than the arm if handwriting at that large a size is possible.

Panel B of fig. 15 shows that changes in writing time also led to changes in spatial variability ($F(5, 15) = 3.94$, $MSe = 1.82 \cdot 10^{-6}$, $p < 0.02$). A Newman-Keuls breakdown suggests that the two shortest target time conditions have levels of spatial variability that are statistically indistinguishable. This variability is, however, substantially larger than the variability in the remaining four conditions. Among these four conditions with longer target times or no target time at all there is again no statistically discernible difference in the level of variability. This simple picture may arise partly because the small sample of subjects limits the power of these comparisons.

Discussion

The primary question addressed by these shape analyses is whether changing writing size, time, or effector leads to changes in the shape of the written product. The answer to this question appears to be yes there are changes in shape, but that they are small. Of course, the characterization 'small' is a somewhat subjective assessment. Besides their size, it is also important to know whether these changes occur systematically across subjects and whether they are characterizable in abstract terms. Given the current technology for assessing shape differences, it is probably only possible to ask the first question after having affirmatively answered the second.

Two patterns of shape change emerge as writing size, time, or effector are varied. Both apply generally for the subjects studied. The first is that the stroke curvature decreases for faster writing and for larger writing: i.e., when writing faster or larger the strokes become increasingly like straight lines. Changing between the hand and the arm does not appear to alter stroke curvature. The second pattern identified involves changes in the relative use of the horizontal and vertical dimensions in the trajectory: i.e., the aspect ratio. As writing becomes larger, more of the increase is in the vertical than in the horizontal dimension. This tendency is substantially more pronounced for writing with the hand than for writing with the arm. Also in the first experiment, the writing in the conditions with no vertical size constraint had a substantially larger vertical component than that in conditions using the same effector and a matched target height.

Of course, it is unlikely that these two patterns of change across conditions summarize all the systematic changes that occurred. In particular, there appear to have been several cases in which single subjects exhibited systematic changes in writing shape, particularly between writing done by the hand and the arm. However, because the patterns did not generalize across subjects, no attempt has been made here to describe or characterize these changes.

One important reason for exploring whether shape is invariant in these experiments is to gain the perspective necessary for the practical problem of interpreting their other results. Clearly, if subjects had maintained temporal invariance in experiment 1 by changing the string that they wrote in the different writing size conditions, we would be mistaken to use the observations of temporal invariance to characterize the motor control system in writing. A similarly inconclusive outcome would result, if we observed that subjects changed writing time in experiment 2 by changing the amount of material written. How then should the shape changes just described modify the conclusions reached in experiments 1 and 2? One possibility is that these changes reflect changes in the movement plan made by the subjects in order to satisfy the experimental constraints in the different conditions. If correct, this interpretation suggests that the results of experiments 1 and 2 should be interpreted cautiously. A second possibility is that, the shape of the intended movement should be seen as being the same in all conditions, in which case, the observed changes in shape may reflect, for example, unanticipated modulations superimposed at a peripheral level because of changes in effector dynamics. These in turn, might be seen to be due to different movement speeds or effectors operating in different parts of their operating range. Under this interpretation, one would be justified in accepting the results of experiments 1 and 2 as evidence about how the control system for writing works. The changes in shape characterized thus far are small enough and sufficiently plausible, interpreted as modulations at the periphery, that I tentatively accept the second of the interpretations above: that the different shapes observed result from the same intended movement.

The conclusion that shape is invariant across large intended changes in writing size and time fits nicely with that of Teulings et al. (1986). Based on an analysis of the vertical positions at stroke endpoints, they conclude that the spatial characteristics of writing are more stable across repetitions and conditions than either temporal or force characteristics.

Along with the comparison of mean shapes, a second reason for analyzing shape was to assess how shape variability changes with changes in writing size, effector, or particularly, with changes in writing time. The emphasis on changes of shape variability with changes in writing time comes from the possibility that shape variability may be least when writing a string in the normal amount of time and that any deviation from this time will lead to an increase in shape variability. Such a pattern of results would support the hypothesis that the relative invariance of writing time observed in experiment 1 across changes in writing size resulted from subjects' attempts to keep shape error near its minimum.

This predicted pattern of results, with minimum shape variability at normal writing time, is present in the mean data of experiment 2. But, although shape variability increases sharply as writing time is reduced from normal, the increase in variability as writing time is increased from normal is too small to be statistically discernible. Thus, if these data give an accurate portrayal of the relation between shape variability and writing time, concerns about increased shape variability would hardly be a strong force working to keep subjects from increasing writing time as target writing size increased. Since the experiment only involved four subjects and a single practised string, it clearly would be worthwhile to obtain additional data on this issue.

The changes in shape variability across changes in writing size and effector are less interesting both because there are no precise predictions to be confirmed or disconfirmed and because the pattern observed appears consistent with what one might reasonably expect. Because shape variability is assessed in the GPA with the trajectories normalized across conditions to a common size, we would expect the shape variability measure to have separate minima in the ranges of hand- and arm-writing. Since the preferred writing movement extent of the hand is smaller than that of the arm, we might expect the minimum in shape variability for the hand to occur at a smaller writing size than that for the arm. Since the hand is generally capable of more precise movements, we might expect the level of shape variability at the minimum for each effector to be less for the hand than for the arm. The results obtained are consistent with all three of these expectations. To fully test these predictions it would be necessary to run an experiment with a wider range of sizes, including both sizes smaller than the 1.0 cm vertical height, which was the minimum in this experiment, and closer spacing of the sizes so that the minima can be located more accurately.

General discussion

This paper has two main results. First, writing time is not invariant across changes in writing size, but increases by a small amount: 0.13 s (8 ms per stroke) per cm of height in the range from 1 cm to 5 cm. This increase is the same for the dominant hand and the dominant arm. This result confirms and extends the previous findings of (Greer and Green 1983; Thomassen and Teulings 1985; Wing 1980). Second, writing time can be varied precisely over a wide range while maintaining size constant and shape relatively invariant. Taken together, these results are incompatible with the view that time has a special role in the control of writing that precludes free variation of writing time and the specific hypotheses that the shape of a stroke is indexed by its duration (Yasuhara 1975) or velocity (Greer and Green 1983).

Although there may not be a structural constraint on writing time, there appears to be a strategic preference for a limited range of writing times. It is probably this preference that explains the relatively small variation in writing time observed in experiment 1 and that has led many previous researchers to the temporal invariance claim (Denier van der Gon and Thuring 1965; Freeman 1914; Hollerbach 1981; Stelmach and Teulings 1983; Stelmach et al. 1984; Yasuhara 1975). Interestingly, however, this preference does not appear to be strong or precisely defined. Instead, this preference appears to take the form of a shallow preference gradient.

The hypothesis that there is a preference gradient for writing times suggests that there is some cost associated with working outside of that range. One candidate, which can be excluded based on the results here, is that changes in speed lead to large, systematic changes in the written product. A related possibility is that shape variability increases as writing is sped up or slowed down. Since relative shape invariance is important for the communicative function of writing, such an increase might be deemed unacceptable. Residual error from a Generalized Procrustes Analysis across the replications in a condition was used to assess this hypothesis. Although shape variability was found to increase sharply as writing time was decreased, the increase in variability as writing time was increased was small and not statistically reliable. Based on this result, it seems unlikely that shape variability is the only cost being avoided by subjects' strategic preference for a limited range of writing times across changes in writing size.

Taken together, the results from these experiments also suggest that effort is not a compelling consideration for subjects in their choice of writing speed. Experiment 1 showed that subjects do not increase writing time proportionately with writing size even though failure to do so increases the effort required. Experiment 2 showed that subjects can modify their writing time much more than they did in experiment 1.

The availability of GPA to estimate and compare the mean shape produced across replications in a condition made it possible in this research to examine more carefully the differences in shape that occur as writing size, effector, or time are changed. These analyses have revealed two aspects of shape that vary systematically across these conditions: the curvature of individual strokes and the relative use of the horizontal and vertical dimensions. Although I do not have a specific theory to explain these effects, it seems plausible that they represent unintended modulations of the motor pattern which occur at the periphery because of changes in movement velocity and extent. Two areas where these differences may be important are automatic handwriting recognition and verification.

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