6 Speed–Accuracy Tradeoffs in Aimed Movements: Toward a Theory of Rapid Voluntary Action

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ABSTRACT

As the speed of rapid aimed movements increases, their spatial accuracy typically decreases. The mathematical form of the speed–accuracy tradeoff depends on the type of movement task being performed. Several alternative hypotheses have been proposed to account for this dependence, including ones that make assumptions about feedback-guided corrective submovements and about the stochastic variability of underlying neuromotor force pulses. A new hybrid class of stochastic optimized–submovement models provides a way to integrate these past accounts in a unified theoretical framework. From the vantage point of this framework, the present chapter reviews the evolution of speed–accuracy tradeoff research and shows how a fresh perspective regarding the properties of elementary movement mechanisms may be obtained.

INTRODUCTION

A fundamental topic in the field of human motor performance concerns the characteristics of rapid aimed movements. Such movements are especially interesting because many physical skills involve sequences of actions during which a person must shift selected parts of the body both quickly and accurately from one place to another in space. Considerable

research has therefore been devoted to studying how movement speed and accuracy are related to each other (Keele, 1968, 1981, 1986).

Significant progress has been made through this research. Investigators have found that precise quantitative tradeoffs occur in rapid aimed movements: As movement speed increases, the spatial accuracy of a movement systematically decreases; conversely, as the spatial accuracy of a movement increases, movement speed systematically decreases. The mathematical form of the speed–accuracy tradeoff depends on certain fundamental details of the movement task at hand (Fitts, 1954; Meyer, Abrams, Kornblum, Wright, & Smith, 1988; Meyer, Smith, & Wright, 1982; Schmidt, Zelaznik, Hawkins, Frank, & Quinn, 1979; Wright & Meyer, 1983; Zelaznik, Mone, McCabe, & Thaman, 1988). Given the nature of the dependence, there is now some hope that a unified theoretical framework for characterizing speed–accuracy tradeoff relations may soon emerge.

The present chapter provides a tutorial review of the research on movement speed and accuracy that has produced this progress. It contains three major parts. First, we outline key aspects of the methodology developed by psychologists, kinesiologists, and neurophysiologists to analyze speed–accuracy tradeoffs. Second, we survey some classic studies regarding the bases of these tradeoffs. Third, we summarize recent empirical and theoretical work conducted in our laboratory to clarify the psychophysical processes underlying various forms of tradeoff relations. In particular, a new hybrid class of stochastic optimized–submovement models (Meyer et al., 1988) is offered here as a way of explaining salient phenomena revealed by both our own experiments and those of other investigators. With these models, one may derive a comprehensive account of numerous details in rapid aimed movements.

Because of constraints on available space, the following review has a necessarily restricted scope. Our main concern is with rapid discrete aimed movements, not slow movements or continuous tracking. Although we recognize the importance of biomechanical and neurophysiological evidence regarding movement speed–accuracy tradeoffs, we primarily consider behavioral data to elucidate them. The range of data under consideration does not include all possible tradeoff relations. We discuss at length how movement speed and spatial accuracy are related to each other, but complementary relations between the speed and temporal accuracy of aimed movements are not addressed fully.[1] For further reviews of these topics, readers may consult a number of other sources (e.g., Georgopoulos, 1986; Hancock & Newell, 1985; Jagacinski, 1989; Jeannerod, 1988; Keele, 1981, 1986; Marteniuk & MacKenzie, 1980; Pew, 1974; Poulton, 1974; Sanes & Evarts, 1984).

[1]The relationship between the speed and temporal accuracy of aimed movements is not necessarily isomorphic with the relationship between movement speed and spatial accuracy. Newell (1980) and his colleagues (e.g., Hancock & Newell, 1985) have claimed that as the average velocity of movements increases, their durations may actually deviate less and less from a selected temporal goal. This claim still has to be assessed systematically, and not all investigators would agree completely with it (cf. Schmidt et al., 1979; Wright, 1983).

METHODOLOGICAL FRAMEWORK

To appreciate the substantive issues that surround movement speed–accuracy tradeoffs, some familiarity with the methodological framework used for pursuing them is essential. This framework embodies two important components: a family of experimental procedures designed to induce observed tradeoff relations, and a set of associated performance measures. The next subsections outline each of these in turn.

Experimental Procedures

Generic protocol. Standard experimental procedures used to study speed–accuracy tradeoffs in discrete aimed movements involve a protocol with a series of individual test trials. On each trial, a marker is first displayed to specify the starting location for an aimed movement, and the subject must align a selected part of the body (e.g., hand) with the location that has been specified. Next, a target is displayed to indicate the desired stopping location for the movement. It may consist of either a point, line, or region placed some distance from the starting location. Then there is a response signal (e.g., auditory tone) that cues the subject to move quickly and stop accurately at the target.[2] The time between the response signal and the initiation of the subject's movement is the *movement latency*. The time between movement initiation and movement termination is the *movement duration*. Both of these times may be measured as a function of certain independent variables. An experimenter may also measure other aspects of the movement, such as its actual endpoint in space and its trajectory from start to stop. Finally, feedback may be provided to maintain motivation and improve performance.

Movement characteristics. If performance were perfect, then following the response signal on each trial, the subject would move in a continuous

[2]In certain special cases, the target may instead represent a location through which the movement must pass quickly at a specified time without stopping there (e.g., Schmidt et al., 1979). An example of this latter type is that of hitting a ball. Although such movements have considerable interest for a complete understanding of speed–accuracy tradeoffs, we do not dwell on them further here.

rapid fashion from the starting location and stop exactly at the target. There would be no hesitations or corrections along the way. However, this is not always what happens in actual practice. Even when the same movement should be made repeatedly, performance may vary from trial to trial, exhibiting both systematic and random fluctuations. On some trials, an observed movement may undershoot the target, whereas on other trials, an overshoot may occur. Also, the trajectories of the movements may contain salient discontinuities, manifesting the occurrence of component submovements (e.g., see Carlton, 1979, 1980; Crossman & Goodeve, 1963/1983; Jagacinski, Repperger, Moran, Ward, & Glass, 1980; Meyer et al., 1988). The properties of this between-trial variability depend on details of how the experimental procedure is implemented.

Alternative movement tasks. Subjects have been given several alternative tasks as part of experiments on movement speed–accuracy tradeoffs. These tasks are defined operationally by the spatial and temporal goals that the subjects must try to achieve with their movements. For example, suppose that a subject has the spatial goal of producing movements whose distances closely approximate a particular target value, D. Then an experimenter may combine this spatial goal with a variety of different temporal goals.

One procedure is to require that the subject produce movements whose durations match a specified target value T, while at the same time making their distances approximate the target value D as closely as possible. We call this a *time-matching movement task* with a spatial constraint (cf. Meyer et al., 1988, who have also called it a "temporally constrained" movement task). As discussed later, the time-matching movement task was the first to be used systematically for exploring speed–accuracy tradeoffs in discrete aimed movements (Woodworth, 1899).

In contrast, a second procedure requires subjects to minimize their average movement durations as much as possible while making their movement distances closely approximate a specified target value D. We call this a *time-minimization movement task* with a spatial constraint (cf. Meyer et al., 1988, who have also called it a "spatially constrained" movement task). The time-minimization movement task was introduced after the time-matching movement task (e.g., Searle & Taylor, 1948) and provides a potentially useful complement. We show subsequently that by comparing results from these two alternative types of task, considerable light can be shed on the processes underlying movement speed–accuracy tradeoffs.

Performance Measures

For the time-matching and time-minimization movement tasks, several aspects of performance must be measured to assess induced speed–accuracy tradeoff relations. Obviously, measures of movement distance and duration are needed. Also, measures are needed of the degree to which the movements achieve their desired spatial and temporal goals. In each case, an experimenter may take several different approaches to meet these needs, and the chosen approach can influence the validity of resulting conclusions.

Movement duration. As indicated already, the duration of a movement is ordinarily defined to be the amount of time between movement initiation (i.e., the moment when a selected part of the body leaves its starting location) and movement termination (i.e., the moment when the stopping location is finally reached). This value may be measured either directly or indirectly. The direct approach involves separately recording both the initiation and termination of each individual movement as a function of time, for example, by extracting them from an explicit record of the full movement trajectory (Meyer et al., 1988). Alternatively, the indirect approach involves having subjects make a series of repetitive movements during a set time interval, and then counting how many movements are made from the moment when the interval starts until the moment when it ends. Given this count, an experimenter can estimate the mean movement duration by dividing the obtained number of movements into the overall length of the time interval (e.g., Fitts, 1954; Woodworth, 1899).

Although experimenters have sometimes taken the indirect approach to measuring movement durations, it seems less desirable than the direct approach. Hesitations having unknown and variable magnitudes may occur between successive repetitive movements, even though subjects are instructed to avoid them. Depending on how such hesitations vary as function of different factors, they can cause spurious biases in the estimated durations, leading to invalid conclusions.

Movement distance. Similarly, there are direct and indirect approaches to measuring movement distance. Using the direct approach, one would explicitly record the starting and stopping locations of each individual movement, treating the difference between them as the movement distance on a trial (e.g., Meyer et al., 1988). In contrast, using the indirect approach, one would simply approximate actual movement distances by assuming that they equal the nominal distance between the starting location and the target (e.g., Fitts, 1954).

Each of these approaches has advantages and disadvantages. The latter (indirect) approach is obviously easier to implement. However, if there are more undershoots than overshoots of the target, or vice versa, then this may introduce systematic biases in subsequent data analyses. Also, important effects associated with between-trial variability of actual movement endpoints may be obscured. Such problems may likewise be en-

countered to some extent when an experimenter tries to measure the distance of a movement directly. For example, background fluctuations in the motor system associated with tremor (Stein & Lee, 1981) and other springlike resonances (Bizzi, Dev, Morasso, & Polit, 1978; Cooke, 1980; Polit & Bizzi, 1979) can make the movement's starting and stopping locations somewhat ambiguous. Nevertheless, the direct approach still has the potential for providing more complete and precise information about movement accuracy than does the indirect approach.

Movement accuracy. Through direct measures of movement distance and duration, the spatial and temporal accuracy of aimed movements may be quantified in several ways. Suppose, in particular, that the target value for a subject's movements on some physical dimension (e.g., distance or duration) is x^*, and that while attempting to achieve this value with each of n movements, the subject produces a set of observed values x_i ($i = 1, 2, \ldots, n$). Then three popular quantitative measures of movement accuracy are available (Schutz & Roy, 1973).

The first of these is *constant error* (CE), the difference between the target value and the mean ($\bar{x}$) of the observed values for a set of movements (i.e., $CE = \bar{x} - x^*$, where $\bar{x} = \Sigma_{i=1}^{n} x_i/n$). CE indicates how close on average the movements come to the target value. With this measure, one can quantify systematic biases that the movements have relative to their spatial and temporal goals. If the biases are small, then CE should be essentially zero. On the other hand, if there is a greater tendency to overshoot than undershoot the target value, then CE would be positive, whereas CE would be negative when there is a greater tendency toward undershoots.

A second complementary measure is *variable error* (VE), the standard deviation of the observed values about their mean (i.e., $VE = [\Sigma_{i=1}^{n} (x_i - \bar{x})^2/(n - 1)]^{1/2}$). With this measure, one can assess the amount of random noise in a distribution of repeated movements. The greater the noise, the greater VE will be. In principle, VE and CE are statistically independent, so experimenters should report both of them, because neither one alone provides a sufficient description of movement accuracy (Schutz & Roy, 1973).

Third, there is *mean absolute error* (AE). It involves subtracting the observed value for each movement from the corresponding target value and then averaging the absolute values of the resulting differences (i.e., $AE = \Sigma_{i=1}^{n} |x_i - x^*|/n$). With this measure, one can assess the average magnitude of a subject's errors regardless of whether they are positive or negative. As the errors increase in magnitude, so will AE. Many past investigators have used AE as their primary dependent variable (e.g., Woodworth, 1899), because it provides a convenient composite indicator of how well a subject has performed.

Unfortunately, AE is also potentially ambiguous and misleading (Schutz & Roy, 1973). It has a high correlation with the sum of CE^2 and VE^2. An experimenter who reports AE without looking separately at these other measures cannot tell to what extent performance has been affected by systematic biases or random noise. Such uncertainty may, in some cases, yield incorrect conclusions.

A similar caveat applies to other summary measures of movement accuracy. When the spatial goal for a set of movements is a target region, accuracy has occasionally been quantified in terms of the relative frequency of target hits (e.g., Fitts, 1954). Like AE, however, the target-hit frequency is actually a joint function of both CE and VE as well as higher-order moments of the movement-endpoint distribution, so it too may yield ambiguous results when reported alone. For this reason, a few investigators have recommended that a large battery of distributional statistics regarding movement endpoints, including not only CE and VE but also skewness, kurtosis, and so forth should be used to characterize a subject's accuracy of performance (e.g., Newell & Hancock, 1984).

HISTORICAL SURVEY

With the preceding methodological points in mind, let us examine more closely some past research on speed–accuracy tradeoffs in discrete aimed movements. For our survey, a period of almost a century is covered, starting right before the year 1900 and progressing up through the late 1980s. The studies included here do not comprise an exhaustive sample of the literature, but they nevertheless illustrate some major trends that have occurred over time. We show how different theoretical accounts of relationships between movement speed and accuracy have emerged from results obtained through alternative movement tasks, and we show how inadequate performance measures have sometimes compromised the validity of these accounts.

Woodworth's Legacy

The individual with whom we begin is R. S. Woodworth, one of the earliest pioneers in the behavioral study of aimed movements. He may be deemed the founding father of research on movement speed–accuracy tradeoffs. Although a few other investigators (e.g., Fullerton & Cattell, 1892) predated Woodworth, their work mainly concerned the effects of movement speed on the accuracy of sensory processes; they did not systematically study how the speed of movement affects the accuracy with which specified spatial and temporal goals (*viz.*, target distance and duration) are attained. Instead, the latter topic served as the primary focus of Woodworth's efforts.

and he contributed the first comprehensive exploration of it. As Woodworth remarked, he sought to establish a psychophysics of human motor performance comparable in aim and scope to sensory psychophysics. Toward this end, his doctoral dissertation, which appeared in 1899, set the standard for many of the subsequent studies of speed–accuracy tradeoff relations.

Research questions. Several important questions guided Woodworth's (1899) work. The first of these concerned whether the spatial accuracy of aimed movements does indeed depend on movement speed. In Woodworth's day, the answer was not entirely clear yet. He was especially impressed by the fact that some rapid movements over long distances seemed to be consistently accurate despite their high speed. For example, in watching Italian construction workers swing a hammer repetitively, he observed that their swings were both fast and nonchalant, but they almost never missed the intended target. The striking quality of the workers' performance stimulated his interest in the sources of movement speed–accuracy tradeoffs and the circumstances under which such tradeoffs occur.

This immediately led Woodworth (1899) to raise a second question concerning what phase of an aimed movement might be affected by its speed. He suggested that rapid movements actually involve two successive phases, which he called *initial adjustment* and *current control*. According to him, the initial-adjustment phase transports a selected part of the body quickly toward a target location; the current-control phase subsequently corrects any errors made along the way, using sensory feedback to reach the target accurately. The issue, then, became one of whether movement speed affects the accuracy of initial adjustment or current control.

Consequently, part of Woodworth's (1899) work focused specifically on feedback processing in error corrections. For example, a third question that occurred to him dealt with what happens when visual feedback is precluded during rapid movements. He reasoned that by eliminating such feedback, one might learn more about the nature of the current-control phase. With this manipulation, his experiments yielded the first measurements of how quickly visual feedback can be processed during current control.

Empirical approach. To obtain relevant data on speed–accuracy tradeoffs in aimed movements, Woodworth (1899) introduced the time-matching movement task with a spatial constraint (*Methodological Framework*). He had subjects sit at a table holding a pencil in their dominant (right or left) hand. Through a slot in the table top, they repeatedly tried to draw lines of a specified length on a roll of moving paper placed beneath the slot. Their rate of movement was paced by a metronome set to produce beats at frequencies ranging from 20 to 200 per min. The subjects completed

one movement back and forth for each beat of the metronome. Their movements were made under two different visual conditions. In one condition, the subjects had their eyes open, and visual feedback was available throughout each moment. In the other condition, their eyes were closed, and no visual feedback was available. Using a ruler, Woodworth measured the accuracy of the movements as a function of the visual condition and movement rate.

Results and conclusions. Some of Woodworth's (1899) results appear in Fig. 6.1, which shows mean absolute error versus movement rate (number of movements per minute) for each visual condition. Under the eyes-open condition (Fig. 6.1, solid curve), the AEs for slow movements were small, but gradually increased with movement rate. Under the eyes-closed condition (Fig. 6.1, dashed curve), the AEs were larger and remained relatively constant regardless of how quickly the subjects moved.

In evaluating these results, Woodworth (1899) assumed that when subjects had their eyes open, performance was mediated by both the initial-

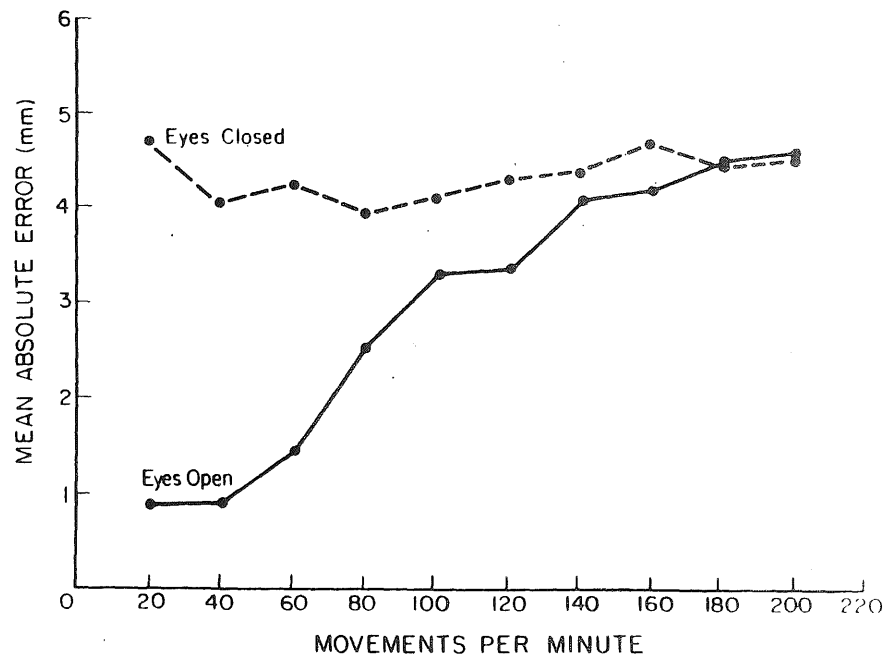

FIG. 6.1. Results from Woodworth's (1899) experiments. (The solid and dashed curves show mean absolute errors plotted versus movement rate, that is, the number of movements per minute, under conditions in which subjects had their eyes open or closed, respectively.)

adjustment and current-control phases of movement. This assumption was consistent with the pattern of errors under the eyes-open condition, where AEs were relatively small except for very fast movements. Moreover, Woodworth assumed that when subjects had their eyes closed, performance was mediated only by the initial-adjustment phase. This assumption was consistent with the pattern of errors under the eyes-closed condition, where AEs were generally larger than those under the eyes-open condition.

On the basis of his assumptions, Woodworth reached several tentative conclusions about the initial-adjustment and current-control phases of rapid aimed movements. He inferred that movement speed did not affect the accuracy of the initial adjustment per se, because under the eyes-closed condition, errors stayed about the same in magnitude as the rate of movement increased. The effect of movement speed on accuracy was attributed instead to the characteristics of current control. In particular, Woodworth inferred that increasing movement speed disrupts error corrections made through visual feedback, because under the eyes-open condition, errors increased in magnitude as the rate of movement increased.

Using the obtained pattern of AEs, Woodworth (1899) also estimated how much time subjects need to process visual feedback. His estimate came from the eyes-open condition, for which the minimum feedback-processing time was taken to be the duration of movements whose errors nearly equalled those in the eyes-closed condition. This yielded a value around 400 msec, about twice the size of an ordinary visual reaction time (cf. Woodworth, 1938).

Evaluation. Taken as a whole, Woodworth's (1899) work on movement speed and accuracy was a veritable *tour de force*. He asked some of the most basic questions about speed–accuracy tradeoffs in rapid aimed movements, collected extensive data to answer these questions, and proposed several interesting hypotheses to interpret the obtained results. His ideas were so insightful that they anticipated much of the research done by subsequent investigators on human motor performance (Keele, 1968, 1981).

Of course, this does not mean that Woodworth's (1899) work was perfect. There were several limitations in his experiments. Because of their instructive nature and ultimate theoretical consequences, these limitations should be kept firmly in mind, even though some reviewers (e.g., Hancock & Newell, 1985; Jeannerod, 1988) have tended to downplay them while focusing instead on the positive contributions that Woodworth made.

The first obvious limitation in Woodworth's (1899) experiments is that they only concerned performance in a time-matching movement task; they did not concern what happens when subjects try to minimize their movement durations while traveling a specified target distance. Perhaps under other circumstances, movement speed might have been found to affect accuracy during both the initial-adjustment and current-control phases.

A second limitation is that Woodworth (1899) did not measure individual movement durations directly. He estimated them indirectly from the number of movements made in time with a metronome. Subjects had considerable freedom to vary the details of the movements, and they may have occasionally gotten out of phase or hesitated along the way. This could have seriously contaminated the results obtained in some cases. Although a few investigators have replotted Woodworth's (1899) data in terms of average movement velocity (e.g., Hancock & Newell, 1985), such extrapolations are not entirely justified, given the vagaries of his experimental technique.

A third limitation is that Woodworth (1899) did not explicitly isolate the initial-adjustment phase of subjects' movements. His conclusions regarding it were based primarily on the endpoints of whole movements in the eyes-closed condition. As mentioned earlier, he assumed that this condition precluded current control by eliminating visual-feedback processing. However, performance there may still have been mediated at least partly through kinesthetic feedback (Crossman & Goodeve, 1963/1983; Keele, 1968). So, from the results obtained when subjects had their eyes closed, one cannot make strong inferences about the initial-adjustment phase per se.

A fourth limitation is that Woodworth (1899) analyzed movement mainly in terms of mean absolute errors. He seldom reported accuracy with respect to constant or variable errors. As a result, the speed–accuracy tradeoff curves derived from his results (*viz.*, Fig. 6.1) could reflect either systematic biases or random variability of subjects' movements. For example, there are two possible reasons why AEs in the eyes-closed condition stayed essentially the same regardless of movement speed. Perhaps the underlying CEs and VEs were both unaffected as movement speed increased, or perhaps the CEs decreased while the VEs increased, such that the joint changes in them roughly canceled each other. If the latter were the case, then Woodworth's (1899) claims about the initial-adjustment phase would be further suspect.

Given the way that the eyes-closed condition was arranged, it seems quite likely that the CEs and VEs obtained for it did change in unknown and uncontrolled ways as a function of movement rate. When subjects had their eyes closed, they received neither on-line visual feedback during the movements nor subsequent knowledge of results about the movement endpoints. Indeed, they could not even see the starting locations or targets of the movements with their eyes closed. Such uncertainties might have easily induced spurious negative correlations between CEs and VEs as movement speed and the memory traces of the targets varied over time. (For other

criticisms of Woodworth's eyes-open and eyes-closed conditions, see Keele & Posner, 1968, and Carlton, 1981).

Dormant Period

Although Woodworth's (1899) experiments had several limitations, a long time elapsed before other investigators followed them up extensively. Only sporadic scientific research on speed–accuracy tradeoffs in discrete aimed movements occurred from 1900 to 1945 (e.g., Garrett, 1922; Philip, 1936). During this dormant period, there were many studies of human motor performance, covering topics such as eye movements (Dodge & Cline, 1901), handwriting (Freeman, 1914), motor memory and movement reproduction (Hollingworth, 1909), sequential timing and rhythm (Langfield, 1915; Stetson, 1905), movement kinematics (Peters & Wenborne, 1936), muscle contraction (Fenn, 1938), and tremor (Travis, 1929). Yet, as these studies progressed, attention gradually shifted away from the issues and results that Woodworth introduced.

Several factors could explain why Woodworth's (1899) work was not followed immediately by more extensive studies of movement speed–accuracy tradeoffs. In light of how much he had discovered, it is possible that he seemed to have resolved most of the mystery surrounding these tradeoffs. As Fitts (1954) has noted, another possibility is that investigators were biased at the time by some emerging prejudices about the nature of movement speed. One case of such bias may be found, for example, in the work of Stetson, a contemporary of Woodworth. From research on rhythm and timing behavior, Stetson (1905) concluded that the duration of fast ballistic movements was independent of how far they traveled in space. He found, in particular, that baton movements made by musical conductors had the same duration regardless of whether their distances were short or long. A similar result was also reported for handwriting (Freeman, 1914). The belief that the duration of ballistic movements did not depend on movement distance then spread among numerous scientists in the motor-control community, later appearing as part of several influential review articles (e.g., Hartson, 1939; Stetson & McDill, 1923). Interest in studying the accuracy of the initial-adjustment phase in aimed movements may have been dampened, consequently, because the prospects for manipulating the duration of this phase to produce speed–accuracy tradeoffs appeared somewhat limited.

Post-World War II Era

During World War II, however, problems associated with the manual operation of military vehicles and weapons stimulated considerable scientific interest in the relationship between movement speed and accuracy (Craik, 1943/1963a, 1944/1963b). Because of these problems, new research was initiated to learn more about the human perceptual-motor system. In this work, considerable effort was devoted to studying continuous tracking and to applying principles of mathematical control theory (Craik, 1947, 1948; Ellson, 1949). Along the way, experiments revealed that tracking behavior often consists of small discrete movements made at a rate of roughly four per sec. Investigators therefore began analyzing such movements more carefully, extending the study of speed–accuracy tradeoffs initiated originally by Woodworth (1899).[3]

Vince. An often-cited product of the latter developments was a study by Vince (1948). Using a procedure that required subjects to track discrete changes in the position of a target line, she replicated many of Woodworth's (1899) original findings, and introduced several new manipulations of visual feedback. Her results substantiated the importance of current control (i.e., feedback-based corrections) for movement accuracy. Still, Vince's study suffered from many of the same limitations as Woodworth's (1899) did. For example, she focused almost exclusively on performance in a time-matching movement task, and she only analyzed movement accuracy with respect to mean absolute errors, not constant or variable errors.

Searle and Taylor. Fortunately, about the same time, two other investigators, Searle and Taylor (1948), conducted an important study that overcame some of these limitations. Their study was one of the first to examine the relation between movement speed and spatial accuracy in a time-minimization movement task. Here subjects controlled a pointer by moving a knob back and forth horizontally. The pointer was supposed to be shifted as quickly as possible so as to position it near a target line whose distance varied from trial to trial. For these movements, Searle and Taylor (1948) carefully examined the initial-adjustment phase by recording and analyzing individual movement trajectories. In their analyses, constant and variable errors, not just mean absolute errors, were measured.

Some of the obtained results appear in Fig. 6.2, which shows VEs for the endpoints of the initial-adjustment phase as a function of average

[3]There was also some applied interest in movement speed–accuracy tradeoffs before World War II. This interest stemmed from industrial time-and-motion studies that were designed to create standardized systems for calculating the rate at which workers could perform various tasks with specified levels of accuracy (Bailey & Presgrave, 1958; Niebel, 1962). However, unlike the subsequent research outlined in our review, these studies did not yield a principled scientific foundation whereby the sources of speed–accuracy tradeoff relations might be better understood.

movement velocity. Searle and Taylor (1948) found that as movement velocity increased, so did the VEs. The monotonic form of the VE versus velocity function foreshadowed results reported by a number of subsequent investigators (e.g., Bainbridge & Sanders, 1972; Schmidt et al., 1979; Wright & Meyer, 1983; Zelaznik, Shapiro, & McColsky, 1981).

This outcome contradicted some of Woodworth's (1899) original conclusions and suggested that the accuracy of the initial-adjustment phase does depend on movement speed. The observed dependence could not be attributed simply to a disruption of error corrections during current control. Thus, it became clear that more exploration was needed regarding the mechanisms underlying speed–accuracy tradeoffs in aimed movements.

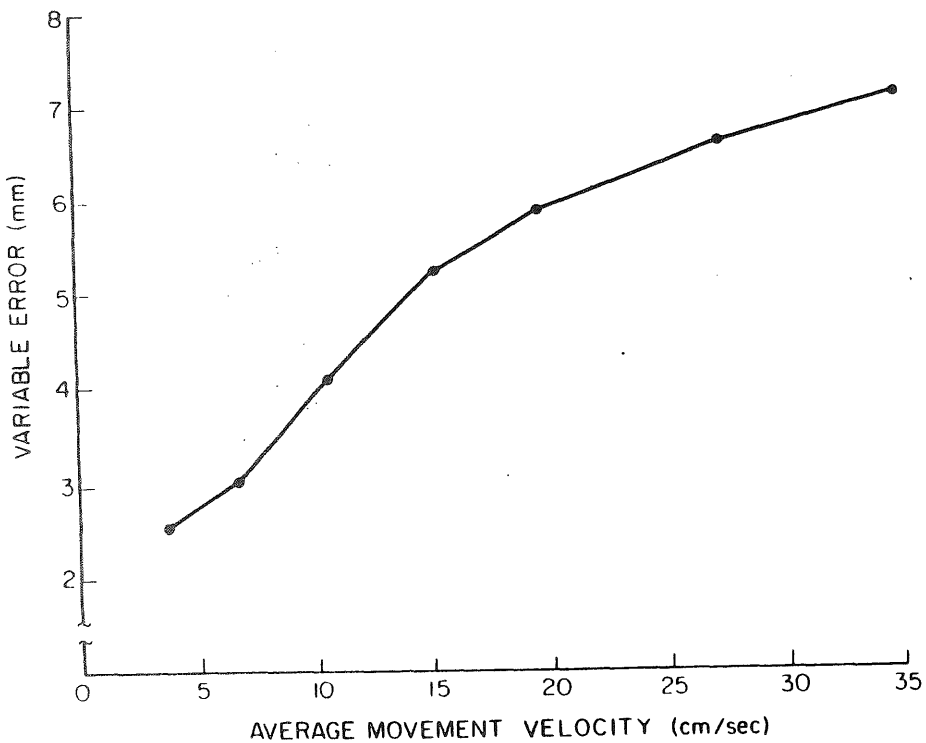

FIG 6.2 Results from Searle and Taylor's (1948) study, which demonstrated an increasing monotonic relation between average movement velocity and variable errors (VE) in the endpoints of the initial-adjustment phase for rapid aimed movements

Brown and Slater-Hammel. The next step along the way was taken soon after, through remarkable, though seldom cited, work by Brown and Slater-Hammel (1949). In their study, subjects again moved a pointer quickly and accurately toward a target line. The key new feature was that the constant and variable errors of subjects' movements were both required to be extremely small; the pointer always had to stop exactly on top of the target line. Movement durations were then measured as a function of target distance, with subjects being encouraged to reach the target as quickly as possible without missing it. Following this measurement, Brown and Slater-Hammel (1949) looked separately at both the initial-adjustment and current-control phases of the obtained movements by analyzing individual movement trajectories.

Some of the results can be seen in Fig. 6.3, which shows mean movement durations (T) from Brown and Slater-Hammel's (1949) study versus the

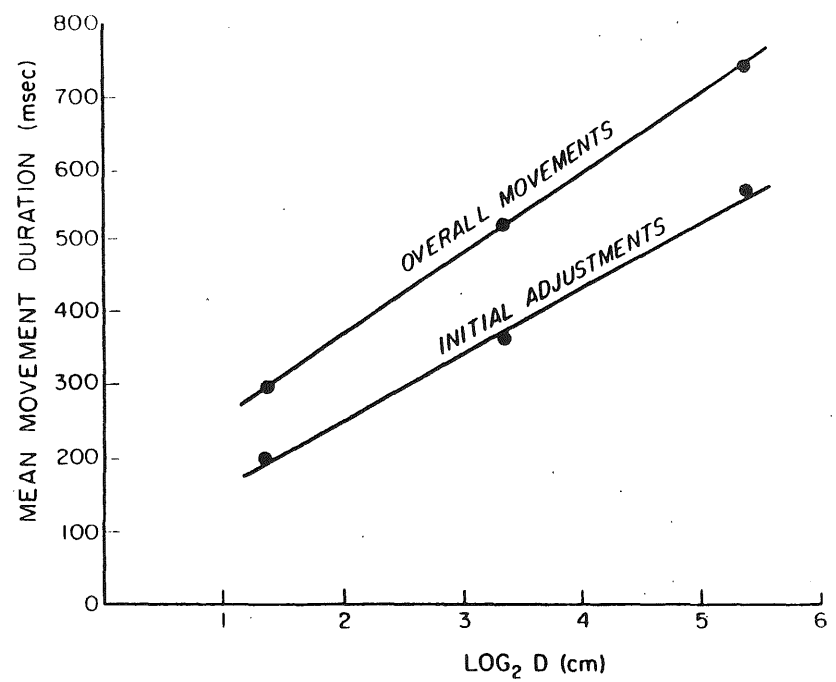

FIG 6.3 Results from Brown and Slater-Hammel's (1949) study, which demonstrated a logarithmic relation between mean movement duration and target distance for initial adjustments and overall movements in a time-minimization movement task

logarithm of the target distance ($\log_2 D$).[4] The data appear separately for initial adjustments and overall movements. There is an almost perfect linear relation between T and $\log_2 D$ (r = .999). In fact, most of the distance effect occurred during the initial-adjustment phase; the duration versus distance function for it essentially parallels the one for the overall movements. This suggests that subjects may have varied the duration of the initial adjustments in order to gain some significant benefit in terms of spatial accuracy. So, contrary to Woodworth's (1899) prior claims, one might further question whether the current-control phase provides the sole basis for movement speed–accuracy tradeoffs.

Another noteworthy fact about Brown and Slater-Hammel's (1949) study is that few citations of it appeared subsequently. There are at least two possible reasons for this turn of events. Although Brown and Slater-Hammel strictly controlled subjects' constant and variable errors, they did not measure movement durations as a function of movement precision per se; their experimental design included only one narrow target width (i.e., a line). They also offered no theoretical account for the underlying relation in their data between mean movement durations and the logarithm of target distance. Indeed, they were apparently unaware of this relation and its potential import.[5] As a result, further work by other investigators soon came to overshadow Brown and Slater-Hammel's (1949) significant empirical discoveries.

Fitts' Law

The next major breakthrough regarding speed–accuracy tradeoffs in aimed movements came 5 years later in 1954 through work by Paul Fitts. His research constituted a direct complement and extension of Brown and Slater-Hammel's (1949) contributions. It provided a more thorough investigation of the functional relation between movement speed and accuracy in a time-minimization movement task, and it introduced a new theoretical framework for explaining how the initial-adjustment phase of aimed movements may mediate their ultimate accuracy.

[4]The choice of the base 2 for the logarithmic distance transformation in Fig. 6.3 is somewhat arbitrary. Other bases could be used instead without changing the linearity of the displayed function; they would only affect the function's slope. We have chosen 2 for the base simply because, in subsequent work (Fitts, 1954), it plays an important theoretical role.

[5]The present analysis of Brown and Slater-Hammel's (1949) results differs from the one that these investigators originally offered. In their initial report, average movement velocity (i.e., speed) was plotted as a power function of target distance, using log–log coordinates. This produced a good fit. Nevertheless, our own plot of mean movement duration versus the logarithm of target distance (Fig. 6.3) fits even better.

Fitts (1954) had subjects move a hand-held stylus rapidly back and forth between two target regions, tapping one of the targets at the end of each movement. The movements were supposed to be made as fast as possible without missing the targets. Movement durations were measured as a function of two independent variables; target distance (D), and target width (W). The manipulation of target width placed an explicit constraint on the constant and variable errors in subjects' movement endpoints. This allowed Fitts to determine more about how the intended precision of a movement affects resulting movement durations.[6]

Logarithmic tradeoff relation. Some of Fitts' results appear in Fig. 6.4, which shows mean movement duration (T) versus a logarithmic transformation of target distance divided by target width [*viz.*, $\log_2(2D/W)$]. Fitts (1954) called this transformation the *index of movement difficulty* (ID). He found that for stylus-tapping movements, there was a strong linear relation between the index of movement difficulty and mean movement duration, as expressed by the following equation:

$$T = A + B \log_2(2D/W), \tag{1}$$

where A and B are constants. The same relation also emerged in studies that Fitts (1954) performed on other types of movement (e.g., pin transfer). His results generalized the speed–accuracy tradeoff obtained by Brown and Slater-Hammel (1949), whose work demonstrated a close linear relation between mean movement duration and the logarithm of target distance (cf. Fig. 6.3). Most important, Equation 1 implies that when an explicit constraint is placed on subjects' variable errors by specifying the target width, W, may influence T systematically over and above the effect that D has.[7]

Information-theory hypothesis. To interpret his results concerning movement speed and accuracy, Fitts (1954) adapted some concepts from information theory, which was popular at the time (Shannon, 1948; Shan-

[6]The original study by Fitts (1954) did have one limitation similar to those of Woodworth's (1899) study. Movement durations were not measured directly; they were measured indirectly be counting the number of repetitive taps made during a 15-sec interval, and then dividing this count into the length of the interval. However, subsequent experiments in which the durations of individual stylus-tapping movements were measured (e.g., Fitts & Peterson, 1964; Fitts & Radford, 1966) revealed that Fitts' (1954) results still held.

[7]Given the constraints imposed through W, theoretical accounts of Equation 1 should deal specifically with the nature of variable errors in rapid aimed movements. One cannot base such accounts solely on the nature of constant errors. Consequently, some recent models (e.g., Bullock & Grossberg, 1988) that attempt to explain the logarithmic tradeoff relation in terms of constant errors must be seriously questioned.

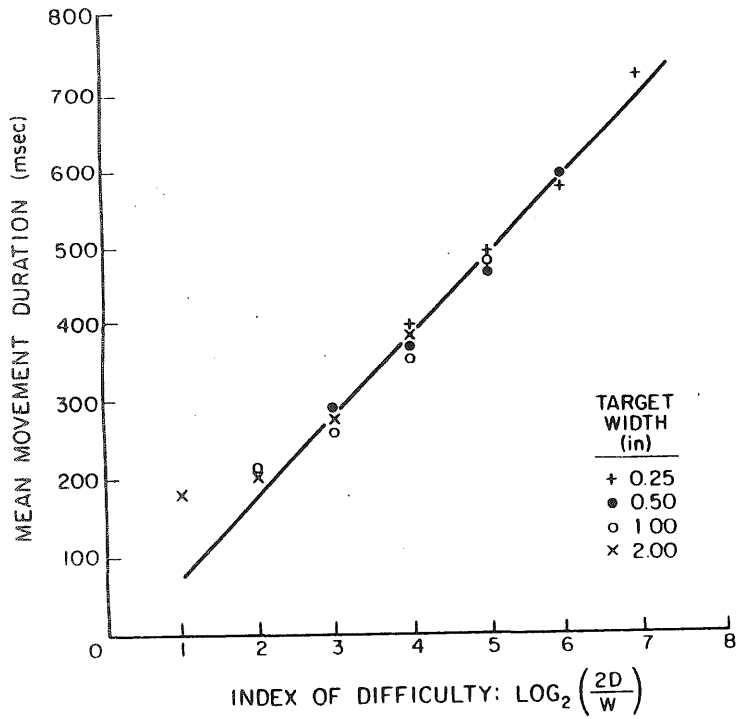

FIG. 6.4. Results from Fitts' (1954, Exp. 1, 1-oz stylus condition) study of stylus tapping, which revealed a strong correlation between mean movement duration (T) and a logarithmic index of movement difficulty [$\log_2(2D/W)$]. (The target distances, D, in this study equaled 2, 4, 8, and 16 in. Each target distance was paired with four different target widths, W, as indicated in the figure.)

non & Weaver, 1949; cf. Miller, 1953). He hypothesized that the human motor system functions like a stochastic communication channel through which information is transmitted in the presence of background noise. This hypothesis could conceivably explain the logarithmic tradeoff relation (Equation 1), assuming that the motor system has a fixed channel capacity, and that subjects move so as to transmit information at the maximum rate possible (Fitts, 1954; Fitts & Peterson, 1964).

For example, suppose one treats reciprocal movements like those in Fitts' (1954) stylus-tapping task as if they embody a waveform composed of a fundamental signal frequency perturbed by white Gaussian noise. Also, suppose one adjusts the noise power (N) to equal half the target width ($W/2$), the signal-plus-noise power ($S + N$) to equal the target distance

(D), and the bandwidth (B) of the signal to equal half the reciprocal of the average movement duration ($1/2T$). Then it has been claimed that at the maximum rate of information transmission, the logarithmic tradeoff relation would emerge (Fitts & Peterson, 1964). This claim is based on a result of Shannon (1948, Theorem 17), who proved that $C = B \log_2[(S + N)/N]$, where C is channel capacity in bits per sec. From Shannon's theorem, Equation 1 can be derived by rearranging terms and replacing B, N, and $S + N$ with the preceding equivalents (i.e., $1/2T$, $W/2$, and D, respectively; however, see Kvålseth, 1979, for a critique of this derivation).

Further studies. Other investigators were therefore quick to test Fitts' (1954) ideas empirically. Further studies soon confirmed the logarithmic speed–accuracy tradeoff (Annett, Golby, & Kay, 1958; Crossman, 1957; Crossman & Goodeve, 1963/1983; Flowers, 1975; Jagacinski, Repperger, Moran, Ward, & Glass, 1980; Knight & Dagnall, 1967; Langolf, Chaffin, & Foulke, 1976; MacKenzie, Marteniuk, Dugas, Liske, & Eickmeier, 1987; Megaw, 1975). The reliability was so great that Equation 1 ultimately became known as *Fitts' Law* (Crossman & Goodeve, 1963/1983; Keele, 1968).

Fitts' Law has been found to hold for many different types of movement, manipulanda, environments, and subject populations (Anderson, 1987; Jagacinski, 1989; Keele, 1968, 1981). The logarithmic tradeoff relation occurs not only in stylus tapping but also in throwing movements (B. Kerr & Langolf, 1977), wrist movements (Crossman & Goodeve, 1963/1983; Jagacinski et al., 1980; Meyer et al., 1988), finger movements (Langolf et al., 1976), leg movements (Drury, 1975), and head movements (Jagacinski & Monk, 1985). Regardless of whether movements involve manipulating a hand-held stylus (Fitts, 1954), rotary handle (Crossman & Goodeve, 1963/1983), joystick (Hartzell, Dunbar, Beveridge, & Cortilla, 1982), computer mouse (Card, English, & Burr, 1978), keyboard (Card et al., 1978), or footpedal (Drury, 1975), the tradeoff relation still continues to be at least approximately logarithmic. Similarly, this relation is exhibited by young adults, children (Wallace, Newell, & Wade, 1978), elderly adults (Welford, Norris, & Shock, 1969), mental retardates (Wade, Newell, & Wallace, 1978), and drugged subjects (Kvålseth, 1977). Some experimenters have even demonstrated the validity of Fitts' Law in underwater environments (R. Kerr, 1973, 1978), aircraft flight (Hartzell et al., 1982), and situations where visual feedback is eliminated (Meyer et al., 1988; Prablanc, Echallier, Komilis, & Jeannerod, 1979; Wallace & Newell, 1983). Given these very robust results, modern systems designers have sought to incorporate Fitts' Law in standards of engineering practice (e.g., Card, Moran, & Newell, 1983; Karger & Hancock, 1982).

Still, some aspects of Fitts' (1954) work met with great doubt. Although his empirical results were easy to replicate, the theoretical framework that he proposed to account for them was not well accepted. Critics felt that the information-theory hypothesis was strained at best, and totally wrong at worst (e.g., Crossman & Goodeve, 1963/1983; Kvålseth, 1979). Consequently, this triggered a search for other ways of explaining the logarithmic speed–accuracy tradeoff.

Deterministic Iterative-Corrections Model

Perhaps the best-known alternative to emerge as a result was developed by Crossman and Goodeve (1963/1983). In evaluating Fitts' (1954) ideas, they doubted that the initial-adjustment phase of rapid aim movements suffered from the noise assumed by the information-theory hypothesis. According to them (Crossman & Goodeve, 1963/1983, p. 283), there is an "empirical difficulty in establishing the existence of the postulated 'noise' . . . . Thus, the supposed 'noise' is apparently not present in the effector systems." So, returning to views that Woodworth (1899) had introduced earlier, Crossman and Goodeve (1963/1983) claimed instead that the current-control phase is the main source of the logarithmic speed–accuracy tradeoff.

Their account of Fitts' Law involves a *deterministic iterative-corrections model*. Under this model, movements intended to hit a target region quickly and accurately consist of several discrete submovements made in rapid succession. These submovements in combination satisfy three basic assumptions.

Assumptions of the model. The first assumption is that each submovement travels a constant proportion (p) of the distance between its starting location and the center of the target. For example, the first submovement would travel a distance pD, the second submovement would travel a distance $pD(1 - p)$, the third submovement would travel a distance $pD(1 - p)^2$, and so forth.

The second assumption is that each submovement takes the same constant amount of time (t) to be completed, regardless of how much distance the submovement travels. Consequently, the duration (T) of the overall submovement sequence should be proportional to the number of submovements in it; if there are n such submovements, then T would equal nt.

The third assumption is that the submovements are guided by sensory feedback. Either visual or kinesthetic feedback might be involved, depending on subjects' ability to see the target and the moving body part. With whatever feedback is available, the submovement sequence supposedly continues until the target region has been reached.

In essence, the iterative-corrections model is completely deterministic. Unlike Fitts' (1954) information-theory hypothesis, it makes no claims about the presence of noise in the motor system during the initial-adjustment or current-control phase of an aimed movement. For a fixed target distance and width, the sequence of submovements would always be the same.[8]

Predictions of the model. The deterministic iterative-corrections model accounts for Fitts' Law in a straightforward manner. It implies that as the ratio of target distance to target width (D/W) increases, the number of submovements required to reach the target region also increases (Crossman & Goodeve, 1963/1983; Keele, 1968). To be specific, let n denote the required number of submovements. Then, according to the model, n approximately equals the index of movement difficulty, $\log_2(2D/W)$. Because each submovement supposedly takes the same constant amount of time (t) to be completed, the overall duration (T) of the submovement sequence should likewise increase logarithmically [i.e., $T = nt = t \log_2(D/W)$].

Furthermore, the model makes some other strong predictions. If it were valid, then one would expect to observe successive discrete corrections in the trajectories of rapid aimed movements toward a given target region. The duration of each submovement should be the same regardless of the target distance and width, and the proportion of the target distance traveled by the initial submovement should be independent of target width (Crossman & Goodeve, 1963/1983; Keele, 1968).

Tests of the model. To test some of these predictions, Crossman and Goodeve (1963/1983) performed experiments in which subjects moved a rotary pointer by making wrist rotations. Such movements were chosen for study because they have several desirable properties. In particular, wrist rotations involve a single axis of motion, making it relatively easy to record and analyze their trajectories, unlike those of complex multidimensional stylus-tapping movements (cf. Fitts, 1954). Also, wrist rotations have relatively low inertia and viscosity, so they may reveal details of

[8]In a later version of the iterative-corrections model, Keele (1968) informally considered the possibility that the distances traveled by the submovements might vary randomly from trial to trial, yielding occasional overshoots as well as undershoots of the target. However, the quantitative predictions that he derived from the model, like those of Crossman and Goodeve (1963/1983), all made the simplifying assumption that the submovements were completely deterministic.

underlying neuromotor control signals more clearly than do some other types of movement.

Crossman and Goodeve's (1963/1983) experiments on wrist rotations provided some support for the deterministic iterative-corrections model. When they plotted the durations of these movements as a function of the index of movement difficulty [$\log_2(2D/W)$], their results exhibited an approximate logarithmic tradeoff relation, consistent with Equation 1. Also, in associated plots of movement velocities, there were sequences of successive peaks whose amplitudes decreased progressively over time as subjects approached the target (e.g., Crossman & Goodeve, 1963/1983). The number of such peaks appeared to vary directly with the index of difficulty, as expected from the model's assumptions about the structure of submovement sequences.

Following the work of Crossman and Goodeve (1963/1983), other investigators were therefore encouraged to study the nature of corrective submovements in more detail. Keele (1968) extended the deterministic iterative-corrections model to incorporate results about the speed of visual-feedback processing. Based on one of his experiments (Keele & Posner, 1968), Keele (1968) estimated that visual feedback takes about 200 msec to be processed (cf. Carlton, 1981; Zelaznik, Hawkins, & Kisselburgh, 1983). Assuming that this time is approximately the same as the duration of a single submovement, he then showed that the model correctly predicts the slope of the logarithmic tradeoff relation (the constant B in Equation 1) found by Fitts (1954) for stylus tapping. This demonstration was sufficiently impressive that the deterministic iterative-corrections model soon became accepted as the best available account of Fitts' Law.

Failures of the model. Since about 1970, however, it has become increasingly clear that the deterministic iterative-corrections model is seriously flawed. Not all of its key predictions have been upheld convincingly. These failures include the following:

1. On frequent occasions involving the time-minimization movement task, there are only one or two submovements per trial, even when the index of movement difficulty is relatively large. For example, Langolf et al. (1976) found numerous cases in which subjects reached a target region with a single submovement, but still exhibited a logarithmic tradeoff relation (Equation 1). The deterministic iterative-corrections model cannot deal with this outcome, because it attributes Fitts' Law solely to systematic variations in the number of submovements made as a function of D/W.

2. Submovements do not have constant durations. Contrary to the deterministic iterative-corrections model, the durations of individual submovements may vary greatly, depending on target distance and width. Langolf et al. (1976) and Jagacinski et al. (1980) have found that initial submovements take significantly more time than subsequent corrective submovements do (cf. Annett et al., 1958). Also, initial-submovement durations appear to increase with target distance and decrease with target width (Jagacinski et al., 1980; Langolf et al., 1976). This suggests that the programming and execution of submovement sequences is considerably more flexible than Crossman and Goodeve (1963/1983) originally assumed.

3. Submovements do not travel a constant proportion of the distance between their starting location and the center of a target region. Jagacinski et al. (1976) have found considerable variability in the proportion of distance traveled by initial submovements, depending on the difficulty of the target. This violates one of the key assumptions made in the deterministic iterative-corrections model to account for Fitts' Law.

4. Subjects sometimes commit errors, missing a target region completely. Fitts (1954) and others (e.g., Fitts & Peterson, 1964; Meyer et al., 1988; Wallace & Newell, 1983) have found that error rates increase with the index of movement difficulty. The deterministic iterative-corrections model cannot explain this, because it assumes that a submovement sequence always continues until the target region is reached. On the other hand, without such continuity, the model would fail to explain the logarithmic speed–accuracy tradeoff.

5. Speed–accuracy tradeoffs in rapid aimed movements are not always logarithmic. They may have other forms instead, disobeying Fitts' Law and violating the main prediction of the deterministic iterative-corrections model (Beggs & Howarth, 1972; Ferrell, 1965; Hancock, Langolf, & Clark, 1973; Howarth, Beggs, & Bowden, 1971; Klapp, 1975; Kvålseth, 1980; Meyer et al., 1982; Schmidt et al., 1979; Sheridan & Ferrell, 1963; Wright & Meyer, 1983).

Other Tradeoff Relations: Power Functions

Some potentially instructive exceptions to Fitts' Law involve power functions having the form

$$T = A + B(D/W)^p, \tag{2}$$

where A and B are non-negative constants, and p is a fractional exponent ($0 < p \leq 1$). In evaluating results of past studies on speed–accuracy tradeoffs for the time-minimization movement task, Kvålseth (1980) discovered that mean movement durations can often be described better by Equation 2 than by a logarithmic tradeoff relation (Equation 1). Other investigators have noted a similar pattern as well (e.g., Ferrell, 1965; Gan & Hoffman, 1988; Hancock et al., 1973; Sheridan & Ferrell, 1963). The exponent of

the best-fitting power function varies a bit from study to study, but usually has a value between .25 and .5. Thus, when T is plotted versus Fitts' (1954) index of movement difficulty, $\log_2(2D/W)$, the data tend to exhibit a slight positive acceleration (i.e., have a U-shape) as D/W increases.[9]

For example, consider Fig. 6.5, which shows some results from one of Crossman and Goodeve's (1963/1983) experiments on wrist rotations. Here we have plotted mean movement durations versus Fitts' logarithmic index of movement difficulty. The solid U-shape function represents a best fitting power function for these data [*viz.*, $T = 187\,(D/W)^{.31}$; $r^2 = .94$]. It comes closer to the mean movement durations than does Fitts' Law [*viz.*, $T = 64 + 83\log_2(2D/W)$; $r^2 = .91$].

A similar pattern can also be seen in Fig. 6.4, where a slight positive acceleration appears in the mean movement durations from Fitts' (1954) stylus-tapping experiment when they are plotted versus the logarithmic index of movement difficulty. The power function that comes closest there is $T = 13 + 165(D/W)^{.36}$. It accounts for about 99% of the variance in the mean movement durations ($r^2 = .988$), whereas Fitts' Law accounts for somewhat less variance ($r^2 = .966$).

The exact form of the tradeoff relation obtained in a given situation may depend on the type of movement task that a subject has to perform (Meyer et al., 1982, 1988; Wright & Meyer, 1983; Zelaznik et al., 1988). As mentioned already, there are different possible tasks, each having its own set of spatial and temporal goals. For the stylus-tapping task in which Fitts' Law was discovered, the subjects' goals were to hit a target region accurately and, given this constraint, to minimize the average movement duration. Under other conditions, however, a different tradeoff may emerge if subjects change their strategy of performance. One might, for example, expect this to happen in a time-matching movement task such as Woodworth (1899) used, where subjects are required to produce movements whose distances and durations both approximate specified target values.

Linear Speed–Accuracy Tradeoff

Consistent with the latter expectation, further evidence that the form of the speed–accuracy tradeoff depends on the subjects' temporal and spatial

[9]In a plot of T versus D/W on an untransformed scale, the obtained curve would exhibit some negative acceleration (i.e., have an inverted U-shape), but the amount of curvature associated with it would be less for a power function whose exponent falls between zero and one than for a logarithmic function. Thus, if T is actually a power function of D/W, and if T is plotted versus the logarithm of D/W, then the logarithmic transformation would overcompensate for the original power function's negative acceleration, yielding an apparent positive acceleration instead.

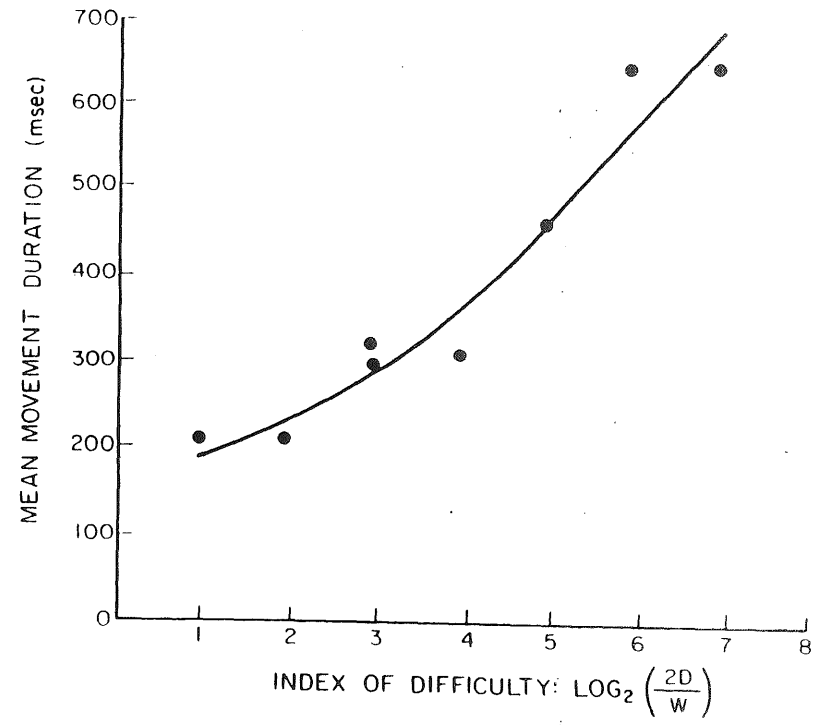

FIG 6.5 Results from Crossman and Goodeve's (1963/1983) study of wrist rotations. (The solid curve represents a power function that relates mean movement durations to the target distance width ratio better than Fitts' Law does For further details, see text.)

goals has been reported by Schmidt et al. (1979). Like some other investigators, they became dissatisfied with the deterministic iterative-corrections model and the reciprocal stylus-tapping task in which Fitts' Law was originally obtained. This inspired them to take a different approach toward studying the relationship between movement speed and accuracy.

Pursuing Woodworth's (1899) lead, Schmidt et al.(1979) used a variant of the time-matching movement task. They had subjects make single aimed tapping movements whose distances and durations were both supposed to match specified target values. The distance and duration of each individual movement were measured directly, unlike in the original studies of Woodworth (1899), yielding better estimates of average movement velocity. Also, Schmidt et al. (1979) examined the constant and variable errors in subjects' performance more carefully.

Some results from one of their experiments (Schmidt et al., 1979, p. 427) appear in Fig. 6.6, which shows variable errors in movement distance

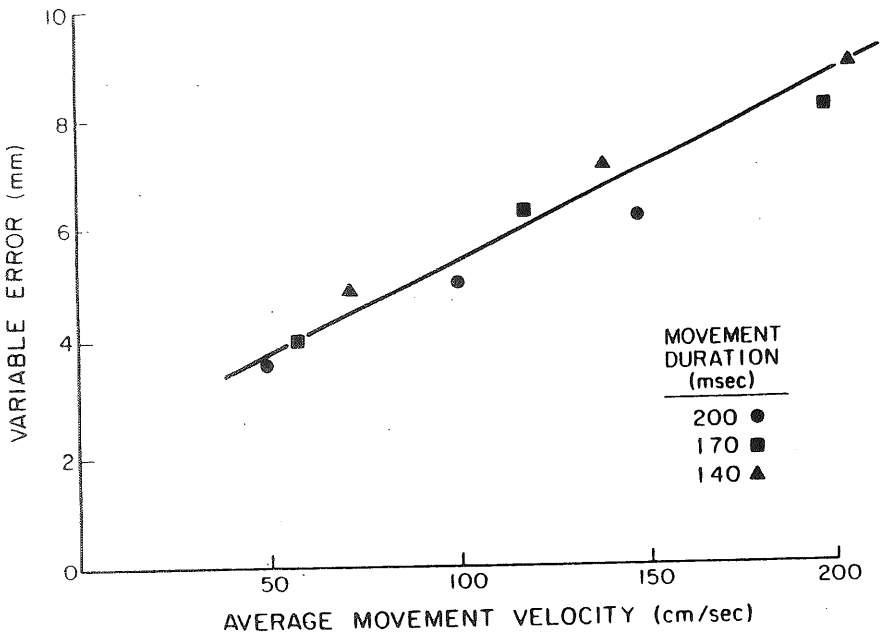

FIG 6 6 Results from a study by Schmidt et al (1979), showing a linear relation between variable error and average movement velocity in a time-matching movement task. (The closed triangles, squares, and circles correspond respectively to movements whose intended durations were 140, 170, and 200 ms. The solid line is the best-fitting linear function.)

versus average movement velocity. A straight line fits these data reasonably well, even though neither of the axes has been transformed logarithmically. We may characterize the obtained linear tradeoff in terms of the following equation:

$$S = A + B(D/T), \tag{3}$$

where S is the standard deviation (variable error) of the movement endpoints in space, D is the mean movement distance, and T is the mean movement duration.

The linear tradeoff relation embodied in Equation 3 differs markedly from Fitts' Law (Schmidt et al., 1979; Wright & Meyer, 1983). In essence, the endpoint standard deviation S may be treated as corresponding to an "effective target width" (W_e). When this correspondence is substituted in Equation 3 and the variables there are rearranged to express T as a function of D/W_e, a nonlogarithmic tradeoff emerges [*viz.*, $T = BD/(W_e - A)$].

For $A = 0$, the latter result is a power function of D/W_e with an exponent of one.[10]

Extending the work of Schmidt et al. (1979), other investigators have found additional impressive evidence of a linear tradeoff relation between the standard deviation of movement endpoints and average movement velocity in the time-matching movement task (e.g., Zelaznik, Shapiro, & McColsky, 1981; Zelaznik et al., 1988). Equation 3 holds not only for stylus-tapping movements but also for wrist rotations (Wright 1983; Wright & Meyer, 1983) and saccadic eye movements (Abrams, Meyer, & Kornblum, 1989). When subjects must make their movement durations match a given temporal goal, this equation appears to have the same functional status as does the logarithmic speed–accuracy tradeoff (Fitts' Law) in the time-minimization movement task.

Impulse-Variability Model

Clearly, the deterministic iterative-corrections model (Crossman & Goodeve, 1963/1983; Keele, 1968), which assumes constant movement endpoints, cannot account for the linear speed–accuracy tradeoff. On the basis of their results from the time-matching movement task and other related paradigms, Schmidt et al. (1979) therefore proposed a new theory called the *impulse-variability model*. It revives some of Fitts' (1954) earlier ideas about noise in the motor system, and adds more detail to them. These details are embodied in the following set of assumptions, which Fig. 6.7 graphically illustrates.

Assumptions of the model. The first assumption is that rapid movements in the time-matching movement task are generated by a pulse of force that has selected amplitude and time parameters. The amplitude parameter (f) supposedly controls the force pulse's height; the time parameter (t) controls its duration. Through these parameters, the force applied to a part of the body may be scaled in amplitude and duration to produce movements having desired average velocities and distances. Evidence of such scaling has been reported by a number of investigators (e.g., Abrams et al., 1989; Armstrong, 1970; Freund & Büdingen, 1978; Ghez, 1979; Ghez & Vicario, 1978; Gordon & Ghez, 1987).

The second assumption is that the amplitude and time parameters are

[10]Because of these differences between Fitts' Law (Equation 1) and the linear tradeoff relation (Equation 3), the former provides a much worse account than does the latter for the results of Schmidt et al. (1979). In particular, applying Fitts' Law to Fig. 6.6 yields $T = -.06 + .038\log_2(2D/W_e)$, $r^2 = .35$, whereas applying the linear tradeoff relation yields $W_e = .22 + .0031(D/T)$, $r^2 = .94$.

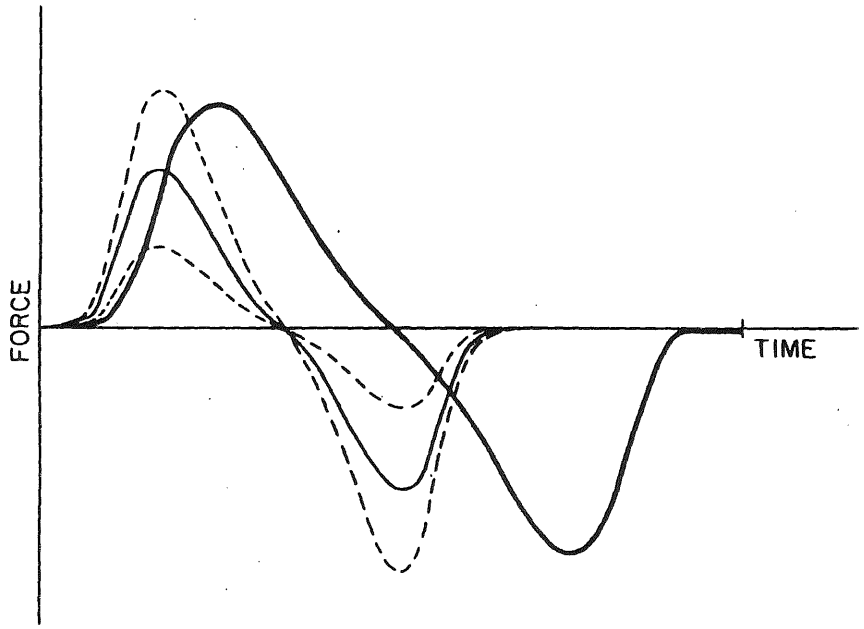

FIG. 6.7 Illustration of assumptions made under one version of the impulse-variability model for rapid movements in the time-matching movement task. (The thick solid curve represents a prototypical force-time function for an individual movement. The thin solid curve and dashed curves represent other functions that may emerge from trial to trial through noise associated with parameters for rescaling the prototype in the force and time dimensions, respectively. From "Models for the speed and accuracy of aimed movements" by D. E. Meyer, J. E. K. Smith, and C. E. Wright, *Psychological Review*, 1982, *89*, 449–482. Reprinted with permission of authors and publisher.)

stochastic variables. A subject supposedly sets them to have certain target values (F and T, respectively), but they may fluctuate randomly about these values because of noise in the motor system. As a result, this produces variability in observed movement trajectories. Some evidence of the noise underlying such variability may be seen, for example, in records of electromyographic (EMG) activity during rapid movements, which typically exhibit agonist and antagonist bursts that fluctuate substantially from trial to trial (Angel, 1974; Bouisset, Lestienne, & Maton, 1977; Brown & Cooke, 1981; Ghez, 1979; Ghez & Gordon, 1987; Hallett, Shahani, & Young, 1975; Lestienne, 1979; Wadman, Denier van der Gon, Geuze, & Mol, 1979; Wallace, 1981).

The third assumption is that the noise in the amplitude and time parameters obeys Weber's Law. Accordingly, their standard deviations (S_f and S_t) would increase proportionally with the parameters' desired magnitudes (i.e., $S_f = K_f F$; $S_t = K_t T$). Movements that must travel long distances at high velocities should, consequently, be more variable than short low-velocity movements. Evidence supporting Weber's Law here has been reported by Abrams et al. (1989), Schmidt et al. (1979), Wright (1983), and other investigators.

Theoretical import. Schmidt et al. (1979) claimed that, taken together, these assumptions of the impulse-variability model account adequately for the linear speed–accuracy tradeoff obtained in their experiments. However, soon after the model was first introduced, some controversy arose regarding whether the model is, in fact, consistent with results from the time-matching movement task. A few investigators have questioned the model's assumptions about the scaling of force pulses (Schmidt, Sherwood, Zelaznik, & Leikind, 1985; Zelaznik, Schmidt, & Gielen, 1986) and about the variability of the force pulse's amplitude and time parameters (Newell, 1980; Newell & Carlton, 1988; Sherwood & Schmidt, 1980). Questions have also arisen over the logic of Schmidt et al.'s (1979) formal theoretical derivations (Meyer et al., 1982; Tsiboulevsky, 1981). Nevertheless, Meyer et al. (1982) demonstrated mathematically that a refined version of the impulse-variability model does yield Equation 3. Some empirical results from saccadic eye movements strongly support their arguments (Abrams et al., 1989). In fact, the notion of impulse variability seems intuitively plausible and may have considerable merit.

SUMMARY

In summary, past research on movement speed and accuracy has involved a variety of tasks from which different speed–accuracy tradeoffs have emerged, including both the logarithmic (Fitts' Law) and linear tradeoff relations. The obtained results have motivated a persistent struggle between two opposing theoretical camps. One camp has claimed that movement speed affects the accuracy of the current-control phase but not the initial-adjustment phase of movement. Its major founder, Woodworth (1899), hypothesized that current control based on visual feedback is disrupted as movement speed increases. In contrast, a second camp has claimed that neuromotor noise increases with movement speed, thereby affecting the accuracy of the initial-adjustment phase. Fitts (1954), the major founder of this camp, characterized the consequences of the noise in terms of

information theory.[11] Despite the formal elegance of his information-theory hypothesis, later investigators rejected it and instead developed further models that attributed speed–accuracy tradeoffs to properties of the current-control phase (Crossman & Goodeve, 1963/1983; Keele, 1968). Yet, recent data have raised more doubts about whether the current-control phase, as embodied in Crossman and Goodeve's (1963/1983) deterministic iterative-corrections model, is the sole mediator of the relationship between movement speed and accuracy. To characterize this relationship more completely, some investigators have therefore revived and extended notions concerning motor noise, as in the impulse-variability model (Meyer et al., 1982; Schmidt et al., 1979).

What none of the preceding contributions does, however, is to provide a good account of Fitts' Law and of the differences in obtained speed–accuracy tradeoffs found for alternative movement tasks. The impulse-variability model has been applied so far only to results from the time-matching movement task, where there is a linear tradeoff relation. It is still unclear how this model, as originally formulated, would explain results from the time-minimization movement task where there is a logarithmic tradeoff relation. A major remaining question concerns whether the logarithmic tradeoff relation stems from the noisiness of the initial-adjustment phase or the structure of error corrections during current control.

The rest of this chapter presents some research that we have conducted to deal with these open issues in more detail. Our goal here is to show how different forms of speed–accuracy tradeoff obtained in alternative movement tasks may be explained with a shared set of principles for the initial-adjustment and current-control phases of rapid aimed movements. Some further efforts toward this goal also appear in Meyer et al. (1988).

STOCHASTIC OPTIMIZED-SUBMOVEMENT MODELS

The theoretical framework for our research is based on a class of *stochastic optimized-submovement models* that we have developed (Meyer et al., 1988; Smith, 1988). These models synthesize and extend ideas introduced previously as part of the deterministic iterative-corrections model (Crossman & Goodeve, 1963/1983; Keele, 1968), impulse-variability model (Meyer et al., 1982; Schmidt et al., 1979), and normative motor-control theory

[11]One might argue that several other investigators (e.g., Brown & Slater-Hammel, 1949; Searle & Taylor, 1948) also contributed significantly to the formation of the second camp (see *Historical Survey*), but their contributions were less fundamental than those of Fitts (1954), because they did not explicitly articulate ideas about the role of motor noise as Fitts did

(e.g., Hogan, 1984; Hollerbach, 1982; Kleinman, Baron, & Levinson, 1970; Nelson, 1983). Their principal domains of application include both the time-matching movement task in which the linear speed–accuracy tradeoff holds, and the time-minimization movement task in which the logarithmic speed–accuracy tradeoff (Fitts' Law) holds.

To illustrate some of the present theoretical framework's main features, we first consider one simple case called the *optimized dual-submovement model*. It applies specifically to the time-minimization movement task and involves either one or two submovements per trial. After assessing this model's goodness-of-fit, we turn to other, more complex, cases involving three or more submovements per trial.

Assumptions of The Optimized Dual-Submovement Model

The optimized dual-submovement model makes several basic assumptions, as illustrated in Fig. 6.8.

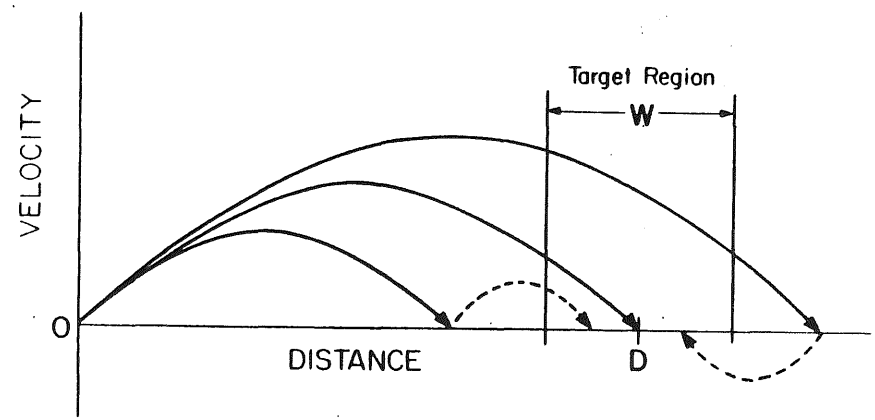

FIG 6.8. Illustration of assumptions made under the optimized dual-submovement model. (The horizontal axis represents movement distance and the vertical axis represents movement velocity. The solid and dashed curves correspond respectively to hypothetical primary and secondary submovements for three trials on which there are movements between an initial home position [distance = 0] and a target region [bounded by vertical lines] whose center is *D* units from the home position and whose width is *W* units. From "Optimality in human motor performance: Ideal control of rapid aimed movements" by D. E. Meyer, R. A. Abrams, S. Kornblum, C. E. Wright, and J. E. K. Smith, *Psychological Review*, 1988, 95, 340–370. Reprinted with permission of authors and publisher.)

Primary submovements. The first assumption is that on each trial of a time-minimization movement task, the subject begins with a primary submovement programmed to hit the center of the target region (Fig. 6.8, middle solid curve). If the primary submovement is successful, then action terminates without further ado.

Motor noise and secondary submovements. A second assumption is that noise in the motor system may affect the primary submovement, causing it either to overshoot or undershoot the target (Fig. 6.8, long and short solid curves). If a miss occurs, then a secondary corrective submovement based on sensory (e.g., visual) feedback is supposedly made (Fig. 6.8, dashed curves). The secondary submovement will usually hit the target, but even it may miss once in a while, because of further motor output variability caused by the noise.

For present purposes, no more corrections are assumed to occur after the secondary submovement. Several considerations motivate this restriction. Although a few investigators (e.g., Crossman & Goodeve, 1963/1983) have reported as many as five submovements per trial, this is not the norm. The findings of other investigators (e.g., Langolf et al., 1976) have revealed that the number of submovements may be rather limited. Also, subjects do commit occasional errors, which suggests that they sometimes stop making submovements without having hit the target. Later, we demonstrate how, even with only one or two submovements per trial, these results may be incorporated into a simple yet plausible account of Fitts' Law.

Effects of noise on submovement endpoints. Third, we assume that the effects of motor noise increase with the velocity of the submovements. In particular, primary submovements are assumed to have endpoints whose standard deviation (S_1) in space is proportional to the average primary-submovement velocity (V_1), as expressed by the following equation:

$$S_1 = KV_1 = K(D_1/T_1), \tag{4}$$

where D_1 is the mean distance traveled by the primary submovements, T_1 is their mean duration, and K is a positive constant. This assumption is consistent with the results of Schmidt et al. (1979) and other investigators (e.g., Wright & Meyer, 1983; Zelaznik et al., 1981), who have shown that variable errors are a linear function of average movement velocity for rapid movements in the time-matching movement task (Equation 3).

Similarly, we assume that the standard deviation (S_2) of secondary-submovement endpoints in space increases proportionally with average secondary-submovement velocity (V_2), as expressed by the following equation:

$$S_2 = KV_2 = K(D_2/T_2), \tag{5}$$

where D_2 is the mean distance traveled by the secondary submovements, T_2 is their mean duration, and K is the same constant as in Equation 4.[12]

Minimization of movement durations. Finally, another key assumption is that the average velocities of the primary and secondary submovements are programmed to minimize the average total movement duration (T). This assumption stems from the demands of the typical time-minimization movement task. Confronted with these demands, subjects presumably try to reach the target region as quickly as possible while attaining some set high proportion of final target hits. To achieve their aim, they must adopt an appropriate strategy for coping with the effects of motor noise. Such a strategy requires making an optimal compromise between the mean duration (T_1) of primary submovements and the mean duration (T_2) of secondary submovements, whose sum determines the average total movement duration (i.e., $T_1 + T_2 = T$).

In particular, the primary submovements should not take too much time. If they are very slow, then this would allow the noise in the motor system to be low, yielding greater spatial accuracy (Equation 4) without a need for secondary submovements. However, it would also tend to overinflate the average total movement duration because of an excessive increase in the mean primary-submovement duration (T_1).

On the other hand, the primary submovements should not take too little time either. If they are very fast, then this would generate lots of noise, causing them to miss the target frequently. As a result, many secondary corrective submovements would then have to be made, and the average total movement duration would again tend to be overinflated because of an excessive increase in the mean secondary-submovement duration (T_2).[13]

[12]More precisely, suppose that a primary submovement ends outside the target region at a point Δ units from the target center. Then secondary submovements made from this point are assumed to have a standard deviation $S_{2\Delta}$ such that $S_{2\Delta} = K\Delta/T_{2\Delta}$, where the mean distance (Δ) and mean duration ($T_{2\Delta}$) of these submovements depend on their starting location. As the endpoints of the primary submovements get farther from the target, Δ and $T_{2\Delta}$ would also increase accordingly. $S_{2\Delta}$ is presumably controlled by adjusting the magnitude of $T_{2\Delta}$ to achieve a desired relative frequency of target hits (Meyer et al., 1988).

[13]Because some of the primary submovements may hit the target without further ado, the mean secondary-submovement duration would include contributions from some trials on which the duration of the secondary submovements is zero. The frequency of such trials depends on the accuracy of the primary submovements, which in turn depends on the mean primary-submovement duration as previously outlined. Thus, the calculation of the mean secondary-submovement duration must take into account a variety of factors, including the relative frequency of primary-submovement target hits, the distances of primary-submovement endpoints from the target when it is missed, and the desired relative frequency of accurate secondary submovements (Meyer et al., 1988).

So, under the optimized dual-submovement model, there is a putative ideal intermediate duration for the primary submovements, and associated with this ideal, there is also an ideal intermediate duration for the secondary submovements. The exact values of these ideals, which depend on target distance (D) and width (W), may be determined jointly through elementary differential calculus (Meyer et al., 1988). By adjusting the mean primary-submovement and secondary-submovement durations accordingly, a subject can simultaneously produce a desired overall relative frequency of target hits while minimizing the average total movement duration.

Predictions of The Optimized Dual-Submovement Model

In essence, the present theoretical framework provides the basis for a normative psychophysics of motor control. It treats subjects as ideal actors who do the best they can, given the demands of their movement task and the limitations imposed by system noise. This perspective parallels one developed in sensory psychophysics, where signal-detection theory has defined optimal behavior for an ideal observer under noisy stimulus conditions (Green & Swets, 1966). Like signal-detection theory, our approach here leads to specific predictions about what the ideal is, and these predictions may then be compared with subjects' actual performance (Meyer et al., 1988). For example, we next consider four quantitative predictions made by the optimized dual-submovement model.

Average total movement durations. The first prediction concerns the average total movement durations (T) in the time-minimization movement task. According to the optimized dual-submovement model, T is a quasi-square-root function of the target distance–width ratio, D/W (Meyer et al., 1988). In particular, Equation 6 should fit very well:

$$T = A + B\sqrt{D/W}, \tag{6}$$

where A and B are non-negative constants.

We therefore expect subjects to obey Fitts' Law approximately, but not exactly. This is consistent with results reported by a number of previous investigators. As mentioned earlier, Kvålseth (1980) has shown that power functions similar to Equation 6 actually come closer to average total movement durations than does a logarithmic function (also see Gan & Hoffman, 1988).[14]

[14]It should be noted, however, that Kvålseth (1980) had no detailed theory to motivate the use of a power function for characterizing movement speed–accuracy tradeoffs. His analyses only involved empirical curve fitting, and in some cases, he had more available parameters than Fitts' Law provides, so it is not too surprising that he obtained improved approximations to available data. In contrast, our approach has a principled theoretical basis; it does not succeed merely through the post hoc addition of extra parameters.

Mean primary-submovement durations. Second, the optimized dual-submovement model predicts that the mean duration of primary submovements (T_1) is a quasi square-root function of D/W (Meyer et al., 1988). In particular, Equation 7 should fit very well:

$$T_1 = A_1 + B_1\sqrt{D/W}, \tag{7}$$

where A_1 and B_1 are non-negative constants ($A_1 < A$, $B_1 < B$). This follows directly from assuming that subjects program their primary submovements optimally. The optimality assumption is supported by results of investigators who have found that primary-submovement durations typically increase as target distance increases and width decreases (e.g., Brown & Slater-Hammel, 1949; Jagacinski et al., 1980; Langolf et al., 1976). In Brown & Slater-Hammel's (1949) study, for example, the durations of primary submovements came very close to having a square-root tradeoff relation with target distance (Fig. 6.3, dashed curve).[15]

Proportion of secondary submovements. A third prediction concerns the proportion of trials (p_2) on which there are secondary submovements. Under the optimized dual-submovement model, we expect p_2 to increase monotonically with D/W. When target distance increases or width decreases, the primary submovements should slow down somewhat (Equation 7), but the decrease in their velocity would not be sufficient to maintain a constant rate of target hits. Instead, given the nature of the optimization process, the primary submovements should miss the target more often, thereby requiring more corrections through secondary submovements. A precise mathematical formula describing the change predicted for p_2 appears in Meyer et al. (1988, Equation 7). It is consistent with results reported by Crossman and Goodeve (1963/1983) and other investigators (e.g., Jagacinski et al., 1980).

Error rates. Fourth, we can also make a prediction about error rates (i.e., the proportion of trials on which a subject finishes without having hit the target region). The optimized dual-submovement model assumes that a trial typically ends after a secondary submovement, even if the target has not been hit yet. Because of this and other related assumptions, the proportion of target misses (p_e) should increase monotonically with D/W. A precise mathematical formula for the predicted increase appears in Meyer

[15]A reanalysis of Brown and Slater-Hammel's (1949) primary-submovement and average total movement durations reveals that square-root functions of target distance fit them almost as well as logarithmic functions do. For primary submovements, $T_1 = 85 + 75\sqrt{D}$; $r^2 = .996$. For overall movements, $T = 183 + 91\sqrt{D}$; $r^2 = .966$. (Note: The time and distance units used in estimating the parameters of these equations are milliseconds and centimeters, respectively).

et al. (1988, Equation 9). It is consistent with error rates reported by Fitts (1954) and other investigators (e.g., Wallace & Newell, 1983).

Tests of The Optimized Dual-Submovement Model

To test these predictions of the optimized dual-submovement model further, we have designed new experiments (Meyer et al., 1988) involving a wrist-rotation task similar to the one used by Crossman and Goodeve (1963/1983). Here the objective has been to analyze movement trajectories precisely for a variety of target distances and widths. From these analyses, the spatial and temporal properties of submovements may be assessed in detail as a function of movement durations, secondary-submovement frequencies, and error rates.

Movement parsing. Our approach uses a set of quantitative rules that parse the position, velocity, and acceleration records of each movement trajectory into component submovements. The parsing algorithm, which is described in Meyer et al. (1988), identifies the beginnings and ends of both primary and secondary submovements, as shown in Fig. 6.9. It also measures the spatial and temporal features of each submovement, including submovement distances and durations. The obtained results thus extend studies of movement kinematics conducted by a number of previous investigators (e.g., Brown & Slater-Hammel, 1949; Carlton, 1979, 1980; Crossman & Goodeve, 1963/1983; Jagacinski et al., 1980; B. Kerr, 1976; Langolf et al., 1976).

Results. Some results of our experiments appear in Fig. 6.10, which shows mean primary-submovement durations (T_1) and average total movement durations (T) versus the square root of the target distance-width ratio. The light and dark circles represent results from twelve different combinations of D and W.[16] The solid and dashed lines represent the corresponding square-root functions that fit the data best. As predicted by the optimized dual-submovement model (Equations 6 and 7), the fit is reasonably good (for average total movement durations, $r^2 = .92$; for mean primary-submovement durations, $r^2 = .90$). In fact, we found a pattern similar to the one reported in Brown and Slater-Hammel (1949), whose data exhibited a montonic, negatively accelerated, relation between target

[16]The values of the target widths (W_t) used in Fig. 6.10 were estimated from subjects' actual performance, rather than simply being set equal to the values chosen by the experimenter (Meyer et al., 1988). We worked with subjective target widths instead of objective ones because subjects may respond to a target somewhat differently than its nominal physical dimensions would dictate (Sheridan, 1979; Welford, 1968).

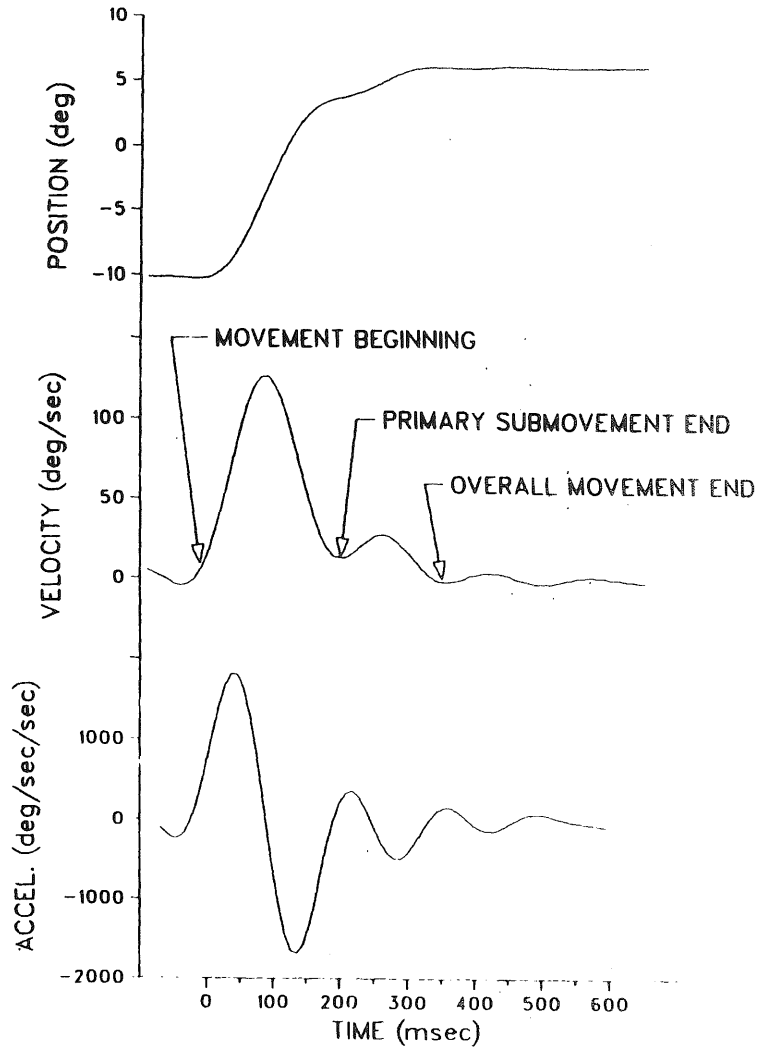

FIG. 6.9 Example of position, velocity, and acceleration records for a representative movement in an experiment on wrist rotations (The target of the movement had an angular distance of 15.8° from the movement's starting location and an angular width of 6.3°. The arrows indicate the beginning of the movement [i.e., primary-submovement start], end of the primary submovement, and end of the secondary submovement [i.e., overall movement end] as identified by the movement-parsing algorithm cited in the text. From "Optimality in human motor performance: Ideal control of rapid aimed movements" by D. E. Meyer, R. A. Abrams, S. Kornblum, C. E. Wright, and J. E. K. Smith, *Psychological Review*, 1988, 95, 340–370. Reprinted with permission of authors and publisher.)

distances and mean primary-submovement durations as well as average total movement durations (Fig. 6.3 and Footnote 15).

More results appear in Fig. 6.11. Here the dark circles represent observed versus predicted percentages of errors (target misses) for the twelve combinations of D and W in our experiment. The dashed line indicates where the circles should have fallen if the observed error percentages had exactly matched the predictions made by the optimized dual-submovement model. The fit is not perfect; some deviations did emerge. However, they were statistically insignificant [$\chi^2(12) < 12.0$; $p > .5$]. As expected, more errors occurred when D/W was large [$F(11,33) = 4.09$; $p < .01$]. So the model again succeeded reasonably well.

We should stress, nevertheless, that some of the model's other predictions have been less successful. In particular, consider Fig. 6.12. Here the

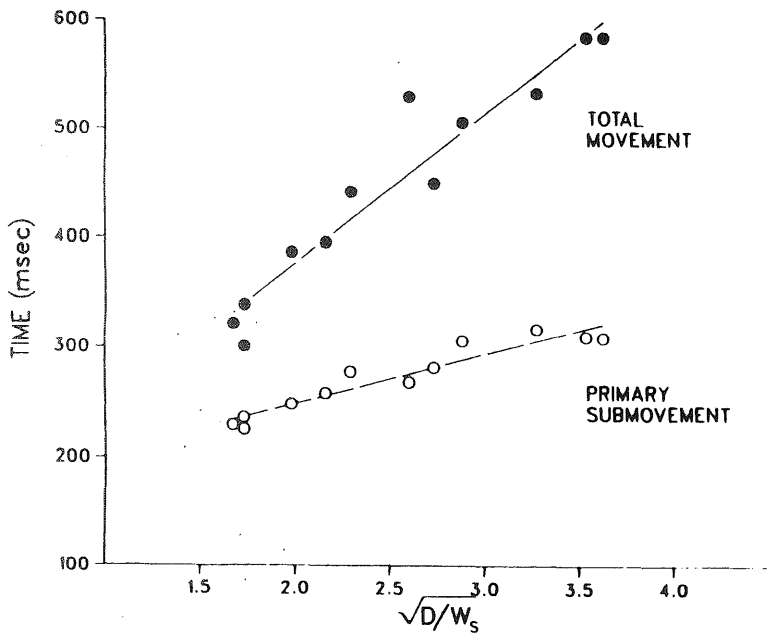

FIG 6.10 Average total movement durations and mean primary-submovement durations versus the square root of target distance (D) divided by subjective target width (W_s) for a time-minimization movement task involving wrist rotations. (The solid and dashed lines represent the fit of predictions made by the optimized dual-submovement model. From "Optimality in human motor performance: Ideal control of rapid aimed movements" by D. E. Meyer, R. A. Abrams, S. Kornblum, C. E. Wright, and J. E. K. Smith, *Psychological Review*, 1988, *95*, 340-370. Reprinted with permission of authors and publisher.)

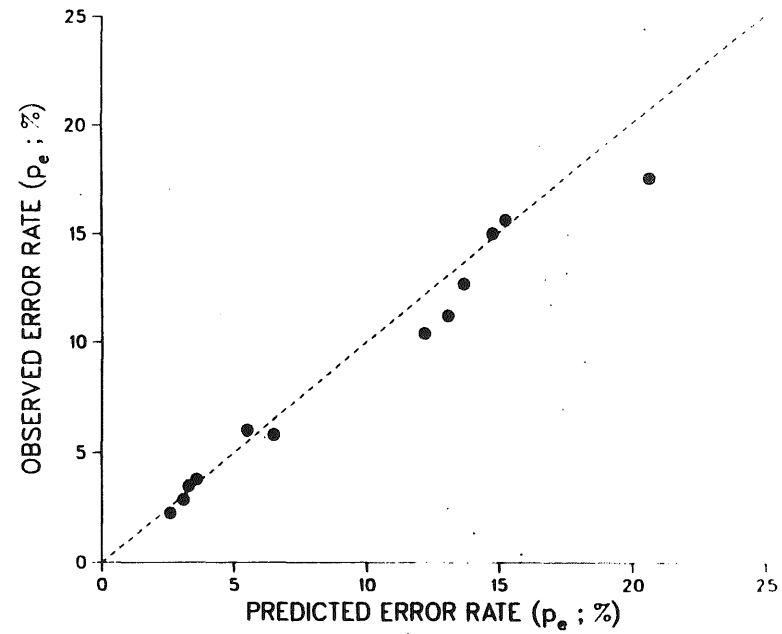

FIG 6.11. Percentages of errors (target misses) from the experiment on wrist rotations in a time-minimization movement task. (The dark circles represent observed and predicted error rates for each of twelve target distance-width combinations. The dashed diagonal line indicates where the circles should have fallen if the predictions of the optimized dual-submovement model fit the observations perfectly. From "Optimality in human motor performance: Ideal control of rapid aimed movements" by D. E. Meyer, R. A. Abrams, S. Kornblum, C. E. Wright, and J. E. K. Smith, *Psychological Review*, 1988, *95*, 340-370.)

dark circles represent observed versus predicted percentages of trials on which secondary submovements occurred for each of the twelve target distance–width combinations. The dashed line indicates where the circles should have fallen if the observed secondary-submovement percentages had exactly matched the predicted ones.

Consistent with these predictions, more secondary submovements tended to occur as D/W increased [$F(11,33) = 11.0$; $p < .001$]. The correlation between the observed and predicted secondary-submovement percentages was high ($r = .96$). Nevertheless, there were also some significant discrepancies [$x^2(12) = 86.0$; $p < .01$]. When the predicted secondary-submovement percentages were low, the observed percentages were even lower. In contrast, when the predicted percentages were high, the observed percentages were even higher.

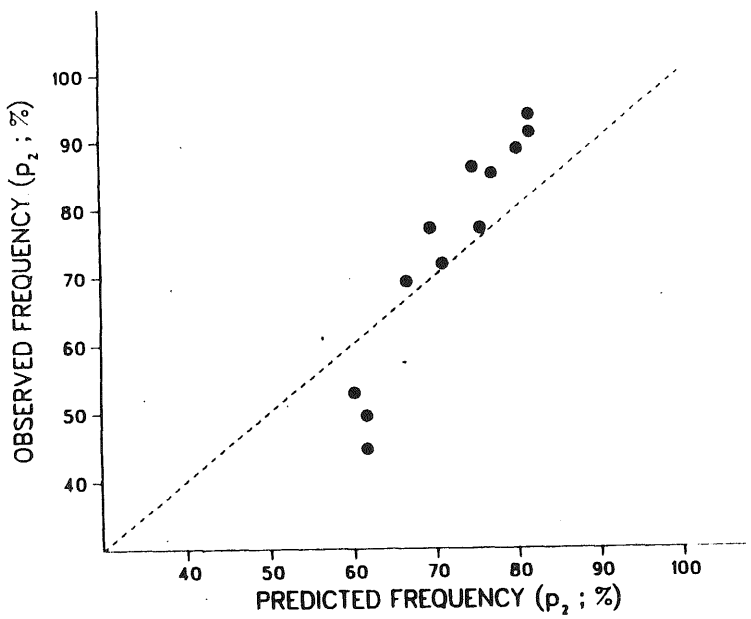

FIG. 6.12. Percentage of trials that yielded secondary submovements during the experiment on wrist rotations in a time-minimization movement task. (The dark circles represent observed and predicted secondary-submovement percentages for each of twelve target distance–width combinations. The dashed diagonal line indicates where the circles should have fallen if the predictions of the optimized dual-submovement model fit the observations perfectly. From "Optimality in human motor performance: Ideal control of rapid aimed movements" by D. E. Meyer, R. A. Abrams, S. Kornblum, C. E. Wright, and J. E. K. Smith, *Psychological Review*, 1988, 95, 340–370.)

Theoretical implications. Our results concerning secondary-submovement percentages might be taken as evidence against the optimized dual-submovement model. Yet we are not ready to reject this model entirely. Its predictions about other dependent variables (e.g., movement durations and error rates) have been confirmed reasonably well. Such confirmation suggests that several of the model's key assumptions are at least approximately correct. In subsequent efforts, we have therefore focused on modifying and extending these assumptions to provide an even better theoretical account of rapid aimed movements in the time-minimization movement task.

Extensions of The Optimized Dual-Submovement Model

One plausible extension of the optimized dual-submovement model concerns the number of submovements assumed to be made in reaching a target region. Although we previously presented an argument for assuming only one or two submovements (i.e., primary and secondary) per trial, perhaps our approach would work even better if it were not so restricted. This has led us to formulate a more general *optimized multiple-submovement model* in which each overall movement toward the target region may include one, two, three, or more submovements.

Additional assumptions. Under the optimized multiple-submovement model, subjects are assumed to make a total of n or fewer submovements per trial. According to this assumption, each trial begins with an initial (primary) submovement toward the target. If by chance the initial submovement hits the target, then action supposedly terminates without further ado. Otherwise, subsequent corrective submovements (i.e., secondary, tertiary, and so on) would be executed, with each one having some intermediate probability of success. The submovement sequence may continue until either the target is hit or n submovements have been completed.

We call n the *maximum submovement number* and treat it as a parameter whose value is independent of target distance and width. If n equals 3, for example, then regardless of D/W, up to 3 submovements may be executed per trial. Alternatively, n could have some other preset value (e.g., $n = 4$). For each value of n, there is a specific case of the optimized multiple-submovement model. Which case best fits a collection of data may, of course, vary from subject to subject and experiment to experiment, depending on particular details of the situation at hand. Thus, n provides the model with a potentially powerful way to characterize a wide range of results.

The remaining assumptions of the optimized multiple-submovement model are similar to those introduced earlier (see *Optimized Dual-Submovement Model*). We still assume that noise exists in the motor system, and that the standard deviations of submovement endpoints increase with the average velocity of the submovements (e.g., Equations 4 and 5). Also, as before, it is assumed that subjects adjust the mean durations of the component submovements so as to minimize average total movement durations while maintaining some desired low error rate.

Additional predictions. On the basis of these assumptions, the optimized multiple-submovement model yields some additional striking predictions about average total movement durations (T) in the time-minimization

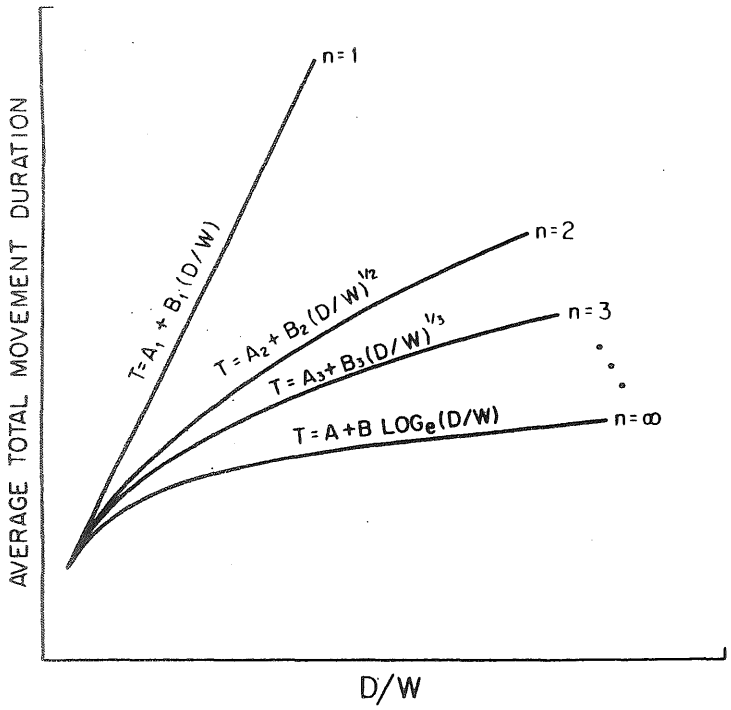

FIG 6.13 Hypothetical average total movement durations (T) versus the target distance - width ratio, D/W (The solid curves show predictions made by the optimized multiple-submovement model as a function of the maximum submovement number, n In each case for which n has a finite value, the predicted relation is a quasi power function whose exponent equals $1/n$ As n grows larger, this relation approaches a logarithmic function, paralleling Fitts' Law)

movement task. We have discovered through mathematical analyses (Smith, 1988) that for each value of the maximum submovement number, T should be a quasi power function of D/W with an exponent of $1/n$ (cf. Equation 2). This pattern is illustrated in Fig. 6.13. For example, if $n = 2$, as under the original optimized dual-submovement model (Fig. 6.8), then the predicted speed–accuracy tradeoff is a quasi square-root function (Equation 6). If instead, $n = 3$, then the predicted tradeoff relation is a quasi cube-root function. And so forth.

Some other details of the predicted tradeoff relations between T and D/W are interesting as well. For any intermediate value of D/W under the

optimized multiple-submovement model, T will generally be less when n is large than when n is small. Increasing the maximum number of submovements enhances the subject's ability to overcome the harmful effects of the assumed motor noise. The resulting benefits are especially salient when D/W itself is large (Fig. 6.13).

Furthermore, as n grows larger and larger, the predicted tradeoff relation approaches a lower bound corresponding to the natural logarithm of the target distance–width ratio [*viz.*, $T = A + B \log_e(D/W)$]. This result, which parallels Fitts' Law, emerges when no restriction is imposed on the maximum number of submovements per trial. So, in essence, the optimized multiple-submovement model encompasses one extreme case (*viz.*, $n = \infty$) that implies Fitts' Law exactly, whereas other intermediate cases (*viz.*, $n < \infty$) approximate it to varying degrees. From a theoretical standpoint, the logarithmic tradeoff relation constitutes the best of all possible solutions that a subject could achieve in terms of time minimization when aimed movements are made through a noisy motor system whose stochastic characteristics satisfy our previous assumptions.

Additional analyses of movement durations. Do subjects actually place a set limit on the number of submovements made in trying to reach a given target region? What particular value of the maximum submovement number (n) yields a predicted speed–accuracy tradeoff that comes closest to real data? The answers to these questions are not necessarily obvious in the light of our previous discourse. As we have seen already, both square root and logarithmic tradeoff relations may fit some data sets reasonably well. So, to obtain further needed insights, we have reanalyzed additional results from Fitts' (1954) original experiments on stylus tapping, which included movements made with a 1-oz stylus and a 1-lb stylus. Given the average total duration (T) of these movements (Fitts, 1954, Table 6.1), our analyses determined the goodness-of-fit obtained with the optimized multiple-submovement model, treating n as a free parameter.

The results appear in Fig. 6.14 (solid and dashed curves). Here the horizontal axis represents the different possible values of n. For each of these, the vertical axis represents the percentage of variance that the optimized multiple-submovement model explains in the average total movement durations reported by Fitts (1954).

We found that the optimized multiple-submovement model with a maximum of three submovements per trial yielded the best fit. It accounts for about 99% of the variance in Fitts' (1954) data from both the 1-oz (Fig. 6.14, solid curve) and 1-lb (Fig. 6.14, dashed curve) stylus conditions. Other cases involving larger or smaller values of n also fit reasonably well, but less so than the three-submovement case. In particular, when we let n be

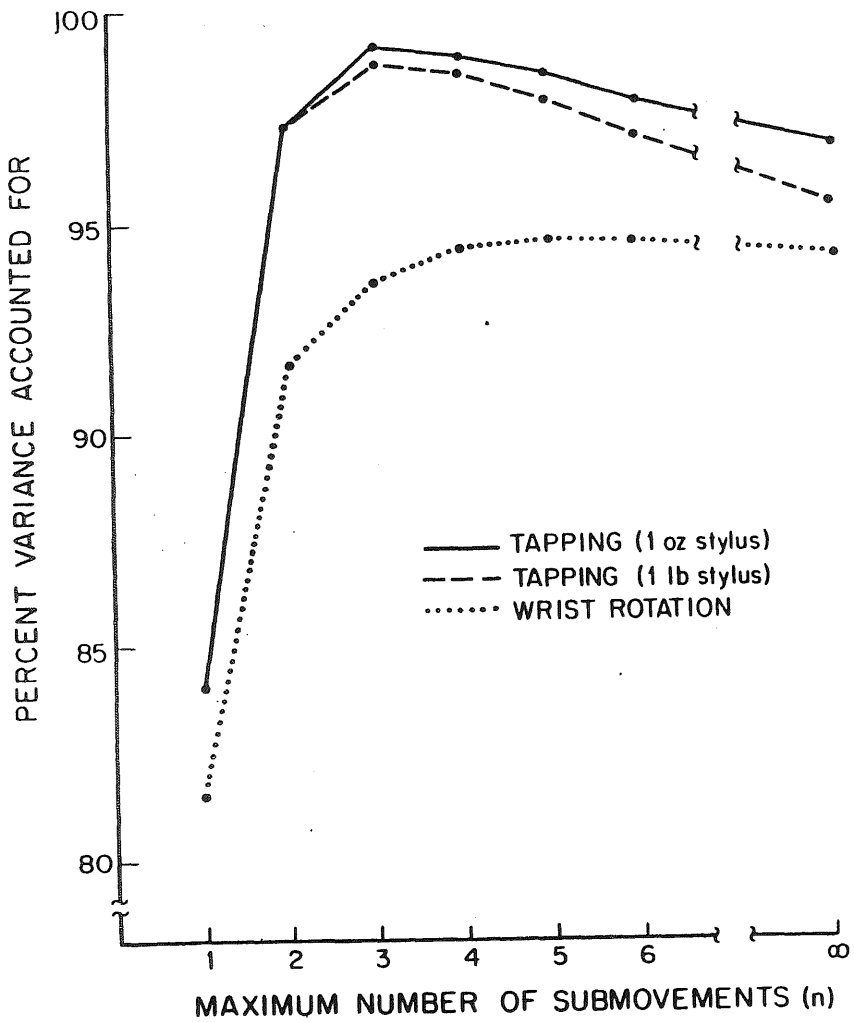

FIG 6.14. Percentages of variance in average total movement durations that the optimized multiple-submovement model accounts for as a function of n, the maximum submovement number. [The solid and dashed curves apply respectively to results from a 1-oz stylus condition and a 1-lb stylus condition in Fitts' (1954, Exp. 1) study of stylus tapping. The dotted curve applies to results from Meyer et al.'s (1988, Exp. 1) study of wrist rotations.]

infinite, corresponding to a logarithmic tradeoff relation (Fig. 6.13), the resulting fit was relatively mediocre (r^2 = .96).[17]

Unfortunately, Fitts (1954) did not record his subjects' full movement trajectories, so we cannot tell whether they really always contained three or fewer submovements per trial. The present support for the three-submovement case of the optimized multiple-submovement model is only indirect. However, we may obtain further evidence about its general validity by looking more carefully at data from the experiments that we have performed on wrist rotations (Meyer et al., 1988).

The relevant data are summarized by the dotted curve in Fig. 6.14. For each possible value of the maximum submovement number, n, it shows the percentage of variance that the optimized multiple-submovement model explains in our subjects' average total movement durations. Unlike for Fitts' (1954) data on stylus tapping, here the most successful cases of the model are those with $n \geq 4$, not $n = 3$.

Several factors could explain this apparent difference between the values of the maximum submovement number that yielded the best accounts for our results (Fig. 6.14, dotted curve) and those of Fitts (Fig. 6.14, solid and dashed curves), respectively. One possibility involves neurophysiological and biomechanical considerations. Perhaps extra fine-grained corrective submovements are easier to make with wrist rotations than with stylus tapping, because of differences between the physical mechanisms underlying these two types of movement.

The value of n that yields the best fit for the optimized multiple-submovement model may also depend on the level of accuracy (target hits) required by an experimenter. If the targets are very narrow, and if an extremely high hit rate is required, then a subject might be induced to make the maximum submovement number relatively large. For example, this could explain why a logarithmic tradeoff relation (Fig. 6.3) fit the results of Brown and Slater-Hammel (1949) so well. As mentioned before, they used very narrow targets (*viz.*, lines) and instructed subjects always to stop their movements right on target. According to our conjecture, such a procedure would encourage a large n value, thereby biasing subjects toward a true logarithmic tradeoff relation (Fig. 6.13), which is what Brown and Slater-Hammel actually found.

Additional analyses of submovement percentages. Some other evidence likewise supports the present theoretical interpretation. In addition to reanalyzing the durations of our subjects' wrist rotations, we have examined

[17]As mentioned previously (Fig. 6.5), the results of Crossman and Goodeve (1963/1983) are also fit best by a power function whose exponent equals about 1/3, consistent with the optimized multiple-submovement model that has $n = 3$.

their movement trajectories again to identify and count submovements at a more microscopic level, using a refined version of the movement-parsing algorithm described earlier (cf. Fig. 6.9). This revealed that on each trial, subjects almost always produced either one, two, three, or four submovements before their activity terminated, consistent with the maximum submovement number of the optimized multiple-submovement model that best fit the average total movement durations (Fig. 6.14).

For example, consider Fig. 6.15. Here the open bars show observed percentages of trials involving one, two, three, and four submovements for a representative target region whose distance–width ratio (D/W) was 9.85. The dashed line shows corresponding percentages predicted by the optimized multiple-submovement model with the maximum number of submovements (n) equal to four. The model's predictions do not fit the data perfectly [$\chi^2(2) = 55.5$; $p < .001$]. Nevertheless, there is good qualitative agreement. Both the predicted and observed percentages exhibit an inverted U-shape pattern, in which a majority of the trials yielded intermediate numbers of submovements.

What caused the less than perfect quantitative fit in Fig. 6.15? One possibility is that our movement parser may have been somewhat inaccurate; perhaps it tended to misclassify single-submovement trials as dual-submovement ones, but tended to misclassify quadruple-submovement trials as involving only three submovements. This could have happened because passive damped oscillations at the end of strong initial submovements are themselves large and may appear much like real voluntary activity, whereas the final member of a submovement quartet is relatively weak and may appear much like residual tremor.

Another possibility is that subjects did not optimize their submovements completely, given a maximum submovement number (n) of 4. If, compared with the ideal, they made the velocities of the initial (primary) submovements too large and the velocities of the tertiary submovements too small, then this could produce discrepancies like those shown in Fig. 6.15. Still, we would infer that subjects came fairly close to ideal performance, because the optimized multiple-submovement model fit the average total movement durations remarkably well (Fig. 6.14) and the submovement percentages at least moderately well (Fig. 6.15).

CONCLUSION

In conclusion, the class of stochastic optimized-submovement models provides a natural extension of previous research on speed–accuracy tradeoffs in rapid aimed movements. Through the theoretical framework embodied by these models, findings from a variety of movement tasks may be ra-

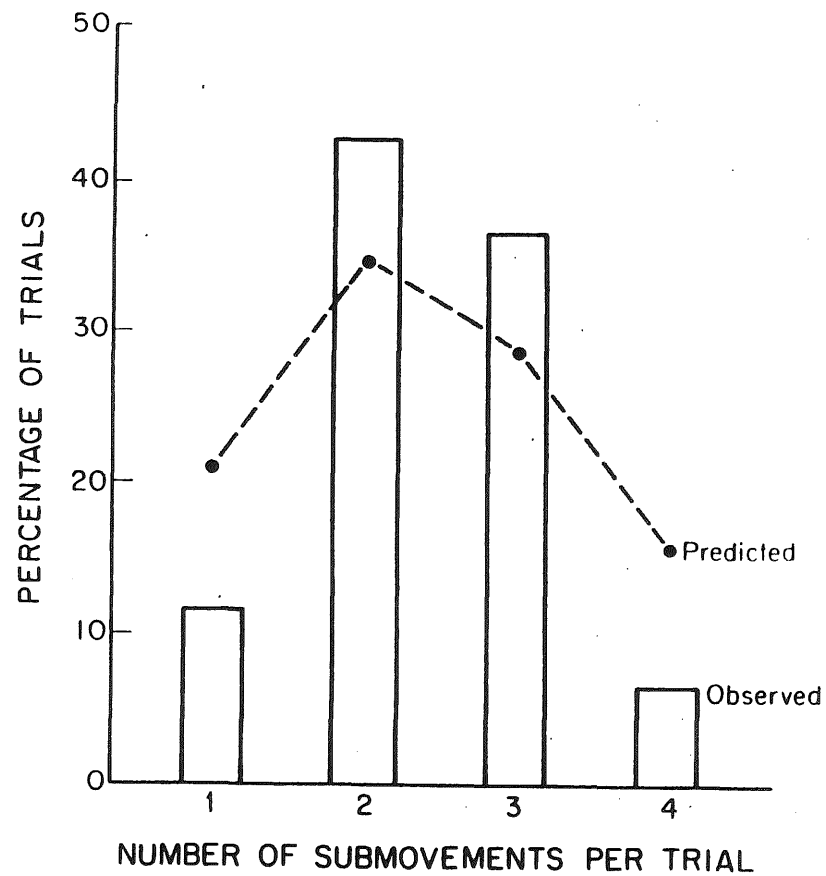

FIG 6.15. Percentages of trials involving either 1, 2, 3, or 4 submovements in the study of wrist rotations conducted by Meyer et al. (1988, Experiment 1) [The results come from a condition in which D/W = 9.82. The solid vertical bars represent observed percentages, and the dashed curve represents predicted percentages for the optimized multiple-submovement model with a maximum submovement number (n) equal to 4.]

tionalized and integrated in a coherent fashion. The linear tradeoff between variable errors and average movement velocity in the time-matching movement task (Equation 3) manifests the underlying random processes assumed as part of this framework. Given the characteristics of these processes and the motor noise associated with them, the logarithmic speed–accuracy tradeoff (Equation 1, Fitts' Law) turns out to be an ideal solution that subjects may adopt for performing successfully in the time-minimization movement task.

As a result, the evolution of research on movement speed and accuracy has a striking parallel with the evolution of research on sensory psychophysics. When early psychophysicists began studying the nature of thresholds, stimulus discrimination, and recognition in sensory processes, they discovered a number of basic phenomena, including Weber's law, Fechner's law, and eventually Stevens' power law (Engen, 1971a, 1971b). These phenomena have been explored through various experimental procedures, culminating in signal-detection theory (Green & Swets, 1966), which explains many of the obtained results in terms of normative statistical-decision rules used as part of a system with noisy inputs. Similarly, studies of speed–accuracy tradeoffs in rapid aimed movements have progressed from a set of fundamental tasks and quantitative tradeoff relations to a theoretical framework involving optimization principles for regulating a noisy motor-output system. Perhaps if Woodworth were still alive today, he would find the present state of affairs at least somewhat satisfying, in that his goal of achieving a mature motor psychophysics may now be within easier reach.

Of course, more research still remains to be done. One important area for future research concerns possible connections between the present stochastic optimized-submovement models and mass–spring models developed previously to characterize slow positioning movements and postural maintenance (Bizzi et al., 1978; Cooke, 1980; Fel'dman, 1966; Polit & Bizzi, 1979; Sakitt, 1980). Under the mass–spring models, movement and posture are controlled by adjustments of the stiffnesses or resting lengths of opposing muscle groups, so as to attain a desired state of equilibrium in their tensions, corresponding to a targeted spatial location. This mode of control raises an interesting open question: how is length–tension programming related to the production of rapid movements through noisy force pulses like those assumed by the impulse-variability and optimized multiple-submovement models?

Perhaps the answer lies in yet-to-be-identified principles of coordination that govern a movement's initial pulselike stage and its final clamping stage, which together propel a selected part of the body toward a new target location and then stabilize it there upon completion of the movement (Bahill & Stark, 1979; Ghez, 1979). As Bahill and Stark (1979) have noted from data on saccadic eye movements, these two stages may not always mesh perfectly with each other, but the neuromotor output signals for each of them do tend to be positively correlated. Consequently, one might expect that similar features of stochastic operation apply in each stage. This similarity, together with considerations of movement optimization, could provide a theoretical bridge for linking models based on force–pulse programming and models based on length–tension programming, thereby resulting ultimately in a unified treatment of fast inaccurate and slow accurate movements.

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